(Weak) Gravitational lensing
(A review)

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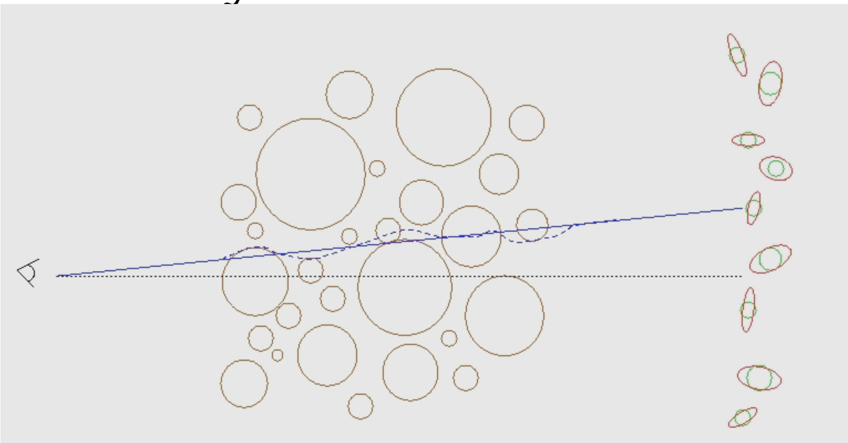
Overview

- Cosmic shear is the distortion of the shapes of background galaxies due to the bending of light by the potentials associated with large-scale structure.
- For sources at $z_s \sim 1$ and structure at $0.1<z<1$ it is a percent level effect which can only be detected statistically.
- Theoretically clean.
- Observationally tractable.



http://mwhite.berkeley.edu/Lensing/

Outline

- Basic lensing theory
- Describing spin-2 fields
- Measuring shear
- Shear statistics
- Tomography & inversion
- Current status of observations
- Future plans
- Weak lensing and dark energy
- Finding clusters with weak lensing
- Simulating weak lensing
- Lensing the CMB

Lensing basics

Recall that photons travel along null geodesics. We can solve for the photon path by extremizing the Lagrangian for force free motion

$$L = \frac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu$$

This leads to the following Euler-Lagrange equations:

$$\frac{dp_\mu}{d\lambda} = \frac{\partial L}{\partial x^\mu} \qquad ; \qquad p_\mu = \frac{\partial L}{\partial \dot{x}^\mu}$$

For the weak field metric

$$ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Phi)d\vec{x}^2$$

Lensing basics (contd)

We obtain to first order in Φ:

$$p_{||}^{-1}\frac{dp_\perp}{d\lambda} = -2\nabla_\perp\Phi\ \dot{x}_{||}$$

which integrates immediately to (becoming cosmological now)

$$d\vec{\alpha} = -2\nabla_\perp\Phi\ d\chi$$

The change in position on a plane perpendicular to the line-of-sight is

$$d\vec{x}(\chi) = r(\chi-\chi')d\vec{\alpha}(\chi')$$

Integrating and dividing by $r(\chi)$ yields the mapping

$$\vec{\theta}(\chi) = -2\int_0^\chi d\chi'\ \frac{r(\chi-\chi')}{r(\chi)}\nabla_\perp\Phi + \vec{\theta}(0)$$

Lensing basics (contd)

Thus the "distortion matrix", which describes the how a ray bundle is modified by its transit through the universe is

$$\frac{\partial \theta_i(\chi)}{\partial \theta_j(0)} \equiv \delta_{ij} + A_{ij}$$

which from before can be written

$$A_{ij} = -2 \int d\chi \; g(\chi) \nabla_i \nabla_j \Phi$$

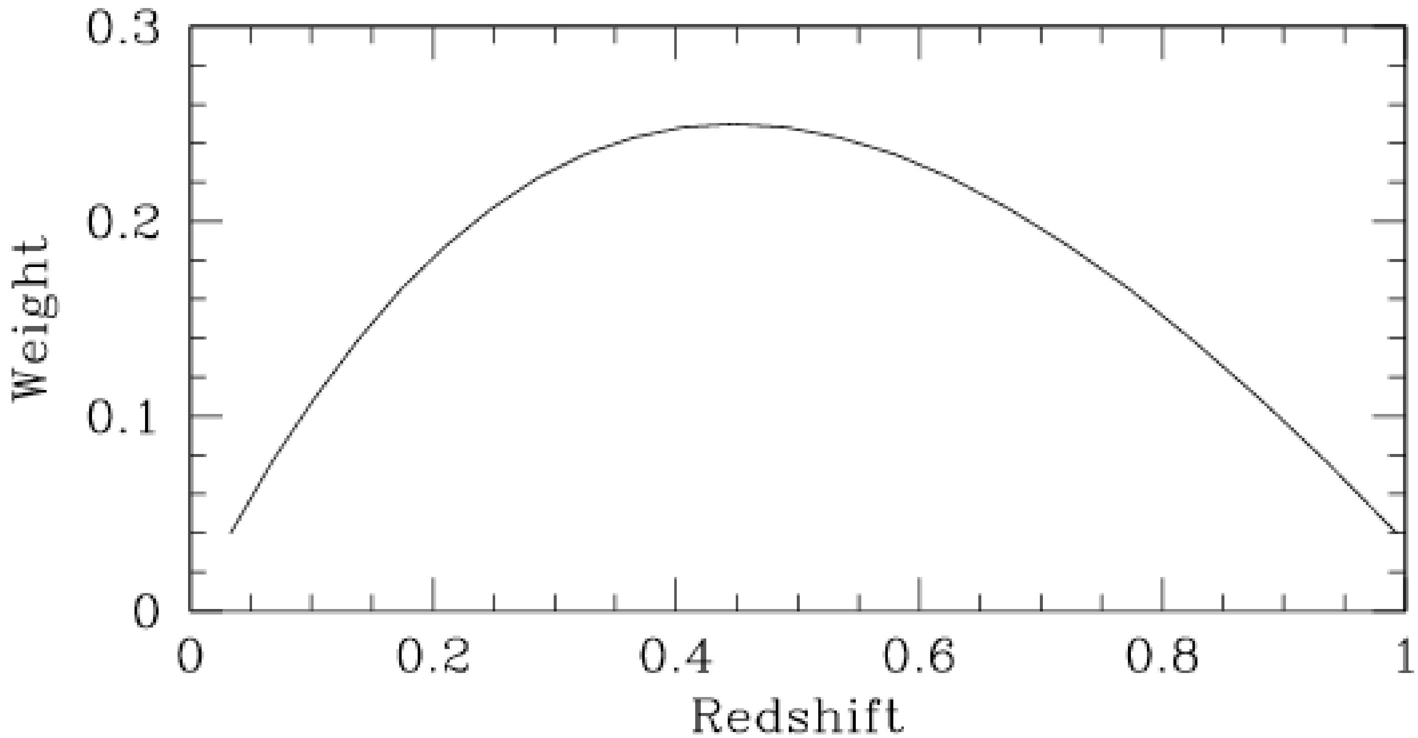
with

$$g(\chi) \equiv \int_{\chi}^{\infty} d\chi_s \; p(\chi_s) \frac{\chi(\chi_s - \chi)}{\chi_s}$$

where in the last line we have specialized to flat space: $r(\chi)=\chi$.

Lensing weight

Lensing is most efficient for structure mid-way between the observer and the source.

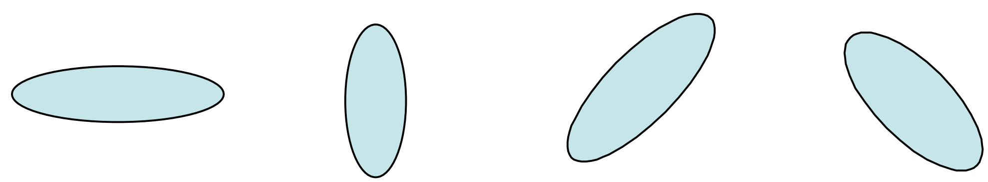


Lensing basics (contd)

The distortion matrix ***A*** is conventionally decomposed as

$$(\mathbf{1}+\mathbf{A})^{-1} = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2-\omega \\ -\gamma_2+\omega & 1-\kappa+\gamma_1 \end{pmatrix}$$

where $\kappa \ll 1$ is the convergence and $\gamma \ll 1$ is the shear.
The rotation, ω, only comes from higher order effects and is much smaller than κ or γ.



$\gamma_1>0$ $\gamma_1<0$ $\gamma_2>0$ $\gamma_2<0$

If the size of the image is not known *a priori* then κ cannot be measured directly. Taking a factor of $(1-\kappa)$ out front of **1+A** we find we can only measure the reduced shear $g=\gamma/(1-\kappa)$.

Lensing basics (contd)

The integral defining **A** should be taken along the perturbed photon path, but the deflection is typically small, so to 1st order we can integrate along a straight line (*Born approximation*).

Then **A** is the second derivative of a projected potential:

$$A_{ij} = -2\int d\chi g(\chi)\nabla_i\nabla_j\Phi \to \nabla_i\nabla_j\phi \to k_i k_j\phi$$

Since κ and γ come from a single potential, ϕ, they can be related via

$$\widetilde{\gamma}_1 = \frac{k_1^2 - k_2^2}{k_1^2 + k_2^2}\widetilde{\kappa} \qquad \text{and} \qquad \widetilde{\gamma}_2 = \frac{2k_1 k_2}{k_1^2 + k_2^2}\widetilde{\kappa}$$

(Kaiser-Squires “method” -- note non-local)

Lensing basics (contd)

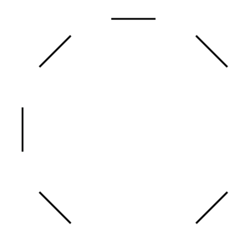
If we relate the potential to the density by Poisson's equation, integrate by parts and ignore the surface term

$$\kappa \simeq \frac{3}{2} H_0^2 \Omega_{\rm mat} \int d\chi \; g(\chi) \frac{\delta}{a}$$

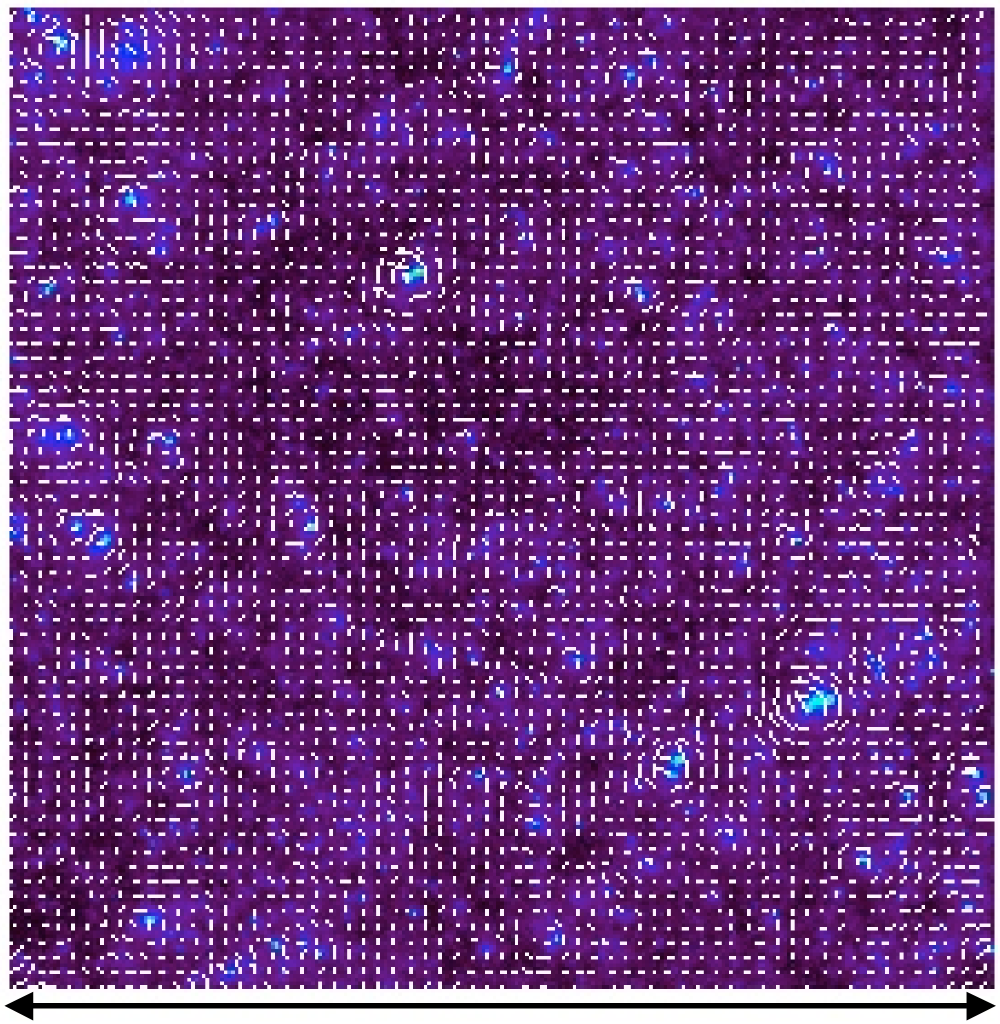
In the Born limit, the convergence is (almost) the projected mass.

It is straightforward to show that a positive, radially symmetric κ leads to a tangential shear:

$$\gamma_1 \propto -\cos 2\phi \qquad \gamma_2 \propto -\sin 2\phi$$



A simulated shear field



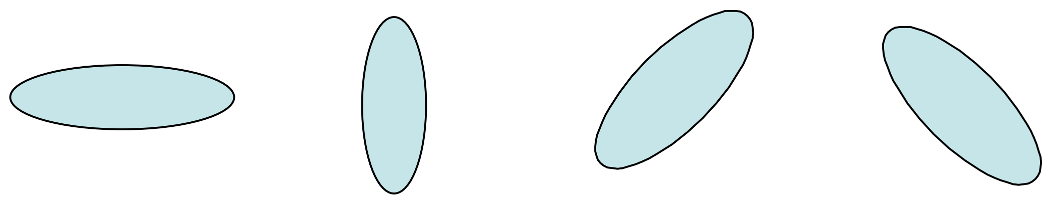
Obvious non-linear structure, with shear tangential about κ peaks of typical size ~1 arcmin.

Filamentary structure erased by projection.

Shear field sampled (regularly) at about the level achievable observationally from deep space based data.

Spin-2 fields

The shearing of images is a spin-2 field. It is useful to spend some time on the description of spin-2 fields.



$\gamma_1 > 0$ $\gamma_1 < 0$ $\gamma_2 > 0$ $\gamma_2 < 0$

Rotating the coordinate system counterclockwise by ϕ changes

$$\gamma_1 + i\gamma_2 \rightarrow (\gamma_1 + i\gamma_2)\, e^{-2i\phi}$$

Under a rotation by π the field is left unchanged.
A rotation by $\pi/2$ changes γ_1 to γ_2 and γ_2 to $-\gamma_1$.

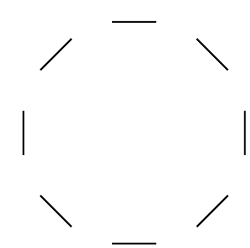
Spin-2 fields (contd)

Keeping track of that phase as we rotate coordinates, the Fourier decomposition can be written in terms of real functions ε and β as

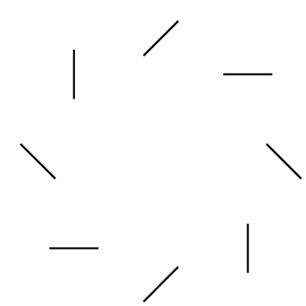
$$(\gamma_1 + i\gamma_2)(x) \equiv \int \frac{d^2k}{(2\pi)^2} \; [\epsilon(k) + i\beta(k)]\, e^{2i\phi_k}\, e^{i\vec{k}\cdot\vec{x}}$$

where ε is parity even and β is parity odd.

The *E*-mode is simply κ -- tangential shear around overdensities.



The *B*-mode is very small for gravitational lensing -- “swirling” around overdensities.



Spin-2 fields (contd)

In the full-sky limit the Fourier transforms become spherical harmonic transforms and Bessel functions become Wigner functions … we gain little by the more general treatment here. For our purposes $k = \boldsymbol{l}$!

For any given line joining two points it is useful to define γ_+ and γ_x wrt the implied axes as

$$\gamma_+ = -Re\left[(\gamma_1 + i\gamma_2)\, e^{-2i\phi}\right] \qquad \gamma_\times = -Im\left[(\gamma_1 + i\gamma_2)\, e^{-2i\phi}\right]$$

where γ_+ is the tangential shear.

2-point functions

If the field is translationally invariant, the Fourier description is useful since <εε> etc are diagonal in *k* or *l*.

$$\langle\epsilon(\mathbf{l})\epsilon(\mathbf{l}')\rangle = (2\pi)^2\delta(\mathbf{l}-\mathbf{l}')C_l^{EE}$$
$$\langle\beta(\mathbf{l})\beta(\mathbf{l}')\rangle = (2\pi)^2\delta(\mathbf{l}-\mathbf{l}')C_l^{BB}$$
$$\langle\epsilon(\mathbf{l})\beta(\mathbf{l}')\rangle = (2\pi)^2\delta(\mathbf{l}-\mathbf{l}')C_l^{EB}$$

P(k) and ξ(r)

For a scalar field, like the matter or galaxy density or the convergence, the correlation function and power spectra form a Fourier transform pair.
For a 2D, isotropic correlation function the relation is a *Hankel transform*:

$$\xi_{2D}(r) \propto \int d^2k \; e^{i\vec{k}\cdot\vec{r}} \, P_{2D}\left(|\vec{k}|\right) \rightarrow \int k \, dk \; P_{2D}(k) J_0(kr)$$

If we work in the flat-sky limit we can rewrite k as l, r as θ and P(k) becomes C_l.

For a spin-2 field the relation is slightly complicated by the presence of the $2i\phi_k$ phase factor in the Fourier transform.

2-point functions (contd)

The shear 2-point function is (by direct substitution)

$$\langle \gamma_i \gamma_j \rangle = \frac{1}{2} \int \frac{\ell d\ell}{2\pi} C_\ell^{EE} \left[\begin{array}{cc} J_0 + c_4 J_4 & s_4 J_4 \\ s_4 J_4 & J_0 - c_4 J_4 \end{array} \right] + \cdots$$

where the arguments are $J_n(l\theta)$ and $\sin(4\phi)$ etc respectively.
In terms of γ_+ and γ_x defined earlier

$$\left\langle |\gamma|^2 \right\rangle = \langle \gamma_+ \gamma_+ \rangle + \langle \gamma_\times \gamma_\times \rangle = \int \frac{\ell d\ell}{2\pi} \left(C_\ell^{EE} + C_\ell^{BB} \right) J_0(\ell\theta)$$

etc.
The shear correlation function is the transform of the *E*-mode power spectrum. Or the shear power spectrum is same as the convergence power spectrum.

2-point functions (contd)

- All other 2-point functions can be written in terms of integrals of the power spectrum times window functions, e.g. shear variance

$$\sigma_\gamma^2 = \int \frac{\ell d\ell}{2\pi} \, C_\ell^{EE} W_\ell^2$$

- Conversion from one quantity to another may not be as straightforward as it could be due to limited range of observations.

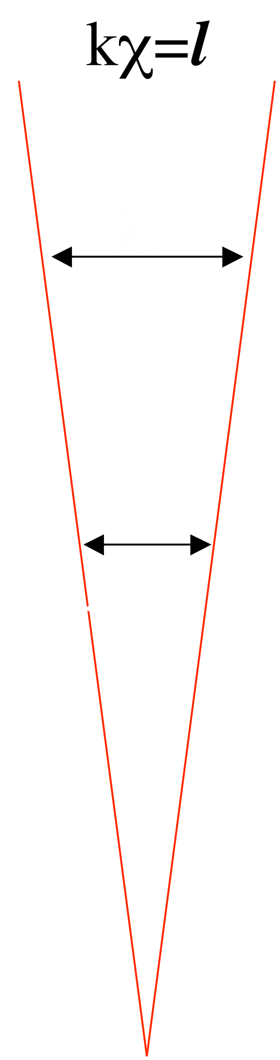
Limber approximation

The Limber approximation allows us to compute the statistics of any projected quantity as an integral over the statistics of the 3D quantity. In its simplest form

$$Q_p(\hat{n}) = \int d\chi \; w(\chi) Q_3(\chi\hat{n})$$

$$\Delta^2_P(\ell) \propto \int \chi d\chi w^2(\chi) \Delta^2_3(k\chi = \ell)$$

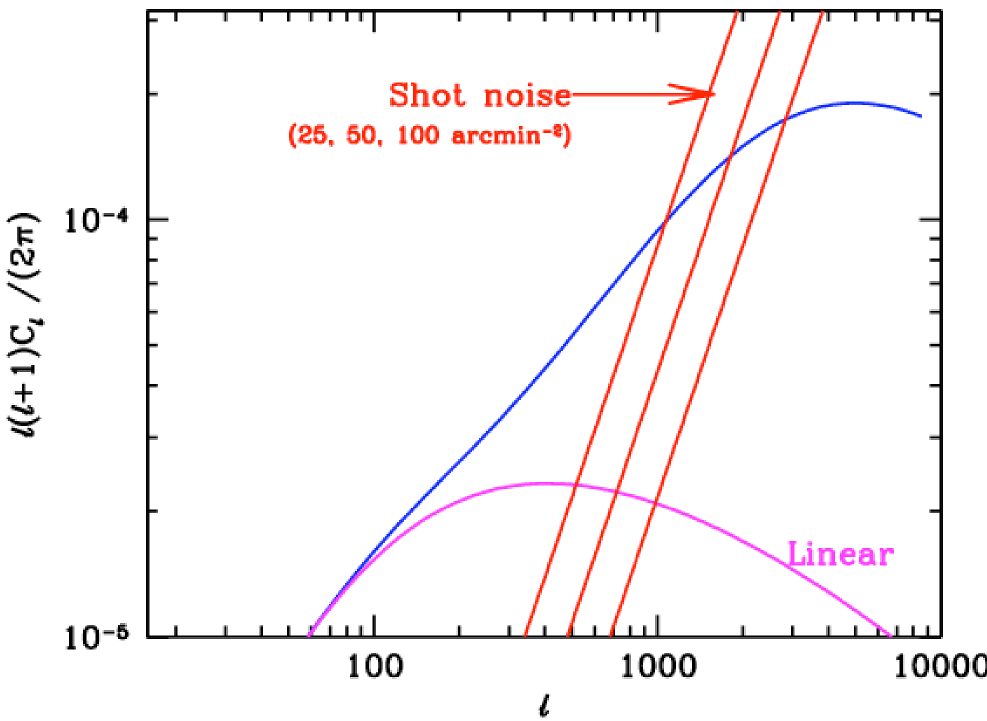
The physical content of the Limber approximation is that, for sufficiently broad *w*, only $k_3=0$ survives and a given angular scales receives contributions from transverse modes.



Lensing power spectrum

Lensing is an obvious candidate for the Limber approximation

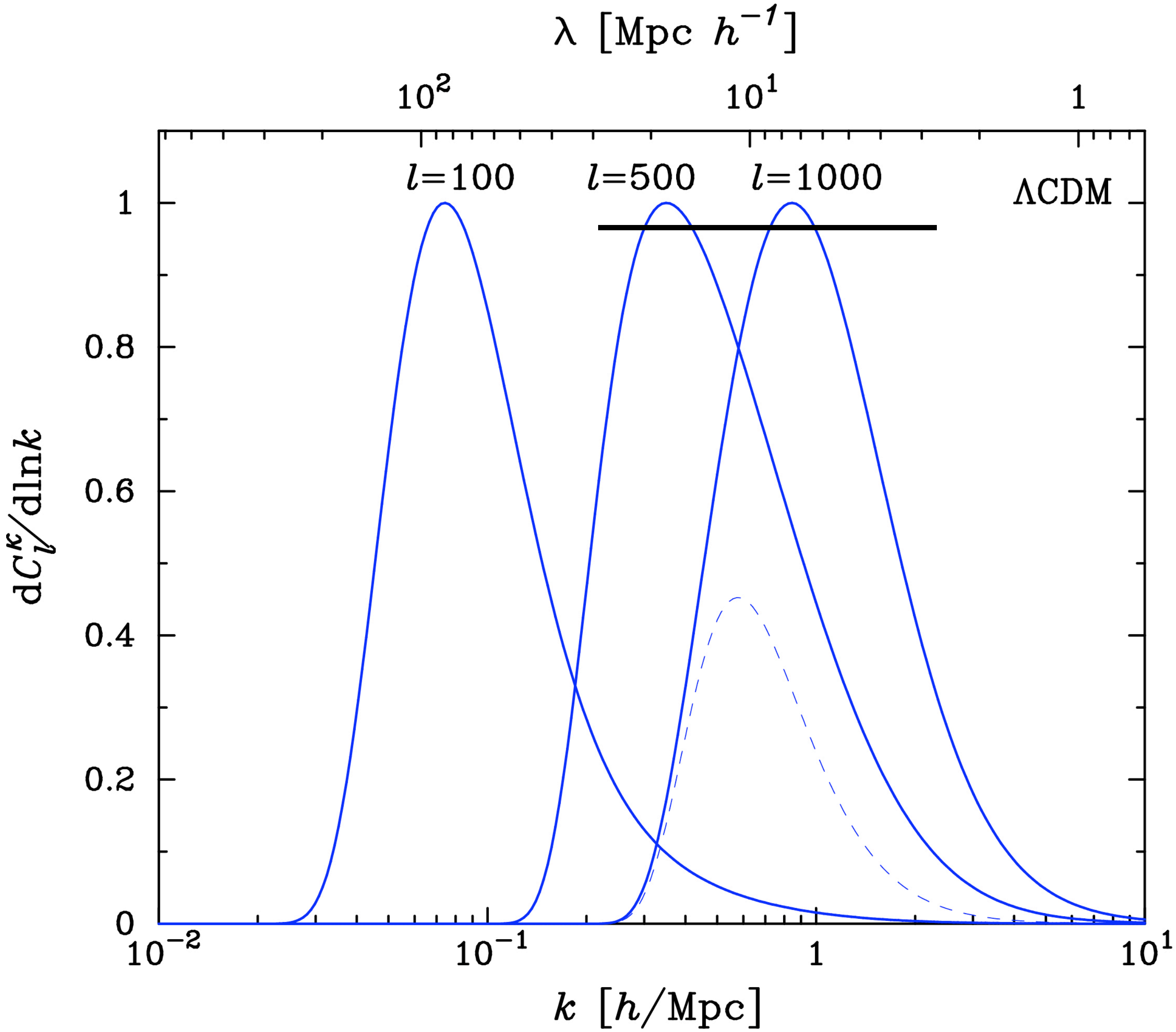
$$\Delta^2_\kappa(\ell) = \frac{9\pi}{4\ell}\Omega^2_{\rm mat}H_0^4 \int \chi'\, d\chi' \left[\frac{g(\chi')}{a(\chi')}\right]^2 \Delta^2_{\rm m}\left(k=\frac{\ell}{\chi}, a\right)$$



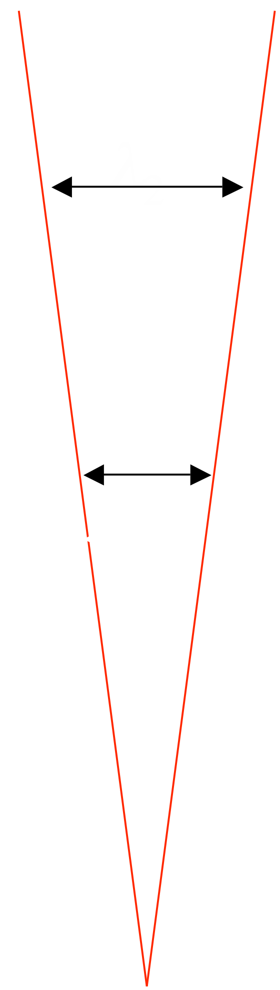
The lensing power spectrum is sensitive to the distance factors, the matter density and the growth of large-scale structure.

Over most of the measurable range it is dominated by non-linear gravitational clustering.

Projection geometry



kχ=***l***



Aperture mass

It is often useful to work with a scalar quantity which is derivable from the shear, is reasonably local and allows a simple E/B decomposition. The "*aperture mass*" is such a quantity. Starting from the relation between κ and γ one can show

$$M_{\rm ap}(\vartheta) = \int^{\vartheta} d^2\theta \ \kappa(\vec{\theta})U(\theta) = \int^{\vartheta} d^2\theta \ \gamma_+(\vec{\theta})Q(\theta)$$

for any compact, compensated filter U where

$$Q(\theta) = \frac{2}{\theta^2}\int_0^{\theta} d\theta' \ \theta' U(\theta') - U(\theta)$$

M_{ap} has a scale beyond which U vanishes.
M_{ap} probes only *E*-modes (replacing γ_+ with γ_x gives *B*-mode)

Aperture mass (contd)

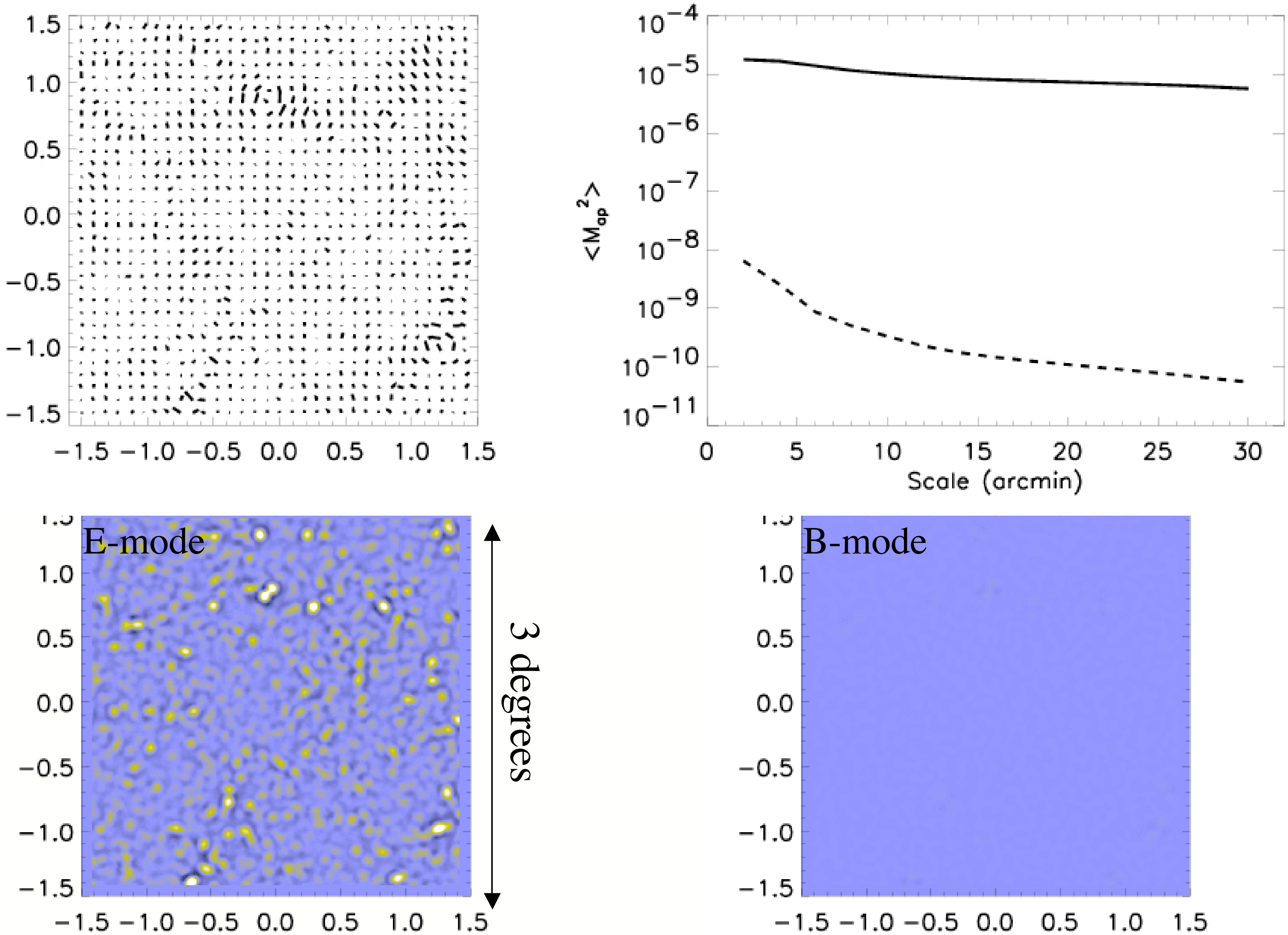
If we pick U to look like a cluster profile, then M_{ap} will be akin to a matched filter.

Generally it is a bandpass filter applied to the κ map.

Advantages:

- M_{ap} can be calculated directly from γ without the need for mass reconstruction.
- Generally M_{ap} is compact in both real and Fourier space (e.g. the kernel for $Var[M_{ap}]$ is J_4)
- M_{ap} decorrelates rapidly beyond the filter scale, so most information is in variance, skewness etc.

Aperture mass (example)



Measuring Shear

Since we don't know *a priori* the positions of galaxies, the deflection is not measurable. However the shearing of shapes or (potentially) the change in sky area is. [Flexion]

Thus we need to work with information about galaxy shapes. The simplest information is the (weighted) moment of inertia:

$$M_{ij} \equiv \frac{\int d^2\theta \; I(\vec{\theta}) w(\vec{\theta}) \delta\theta_i \delta\theta_j}{\int d^2\theta \; I(\vec{\theta}) w(\vec{\theta})}$$

Under A_{ij} the moment of inertia transforms as

$$M^{\rm img} = (1+A) M^{\rm src} (1+A)$$

Measuring shear (contd)

If we define an ellipticity from the 2nd moments

$$e_1 + ie_2 \equiv \frac{M_{11} - M_{22} + 2iM_{12}}{M_{11} + M_{22}}$$

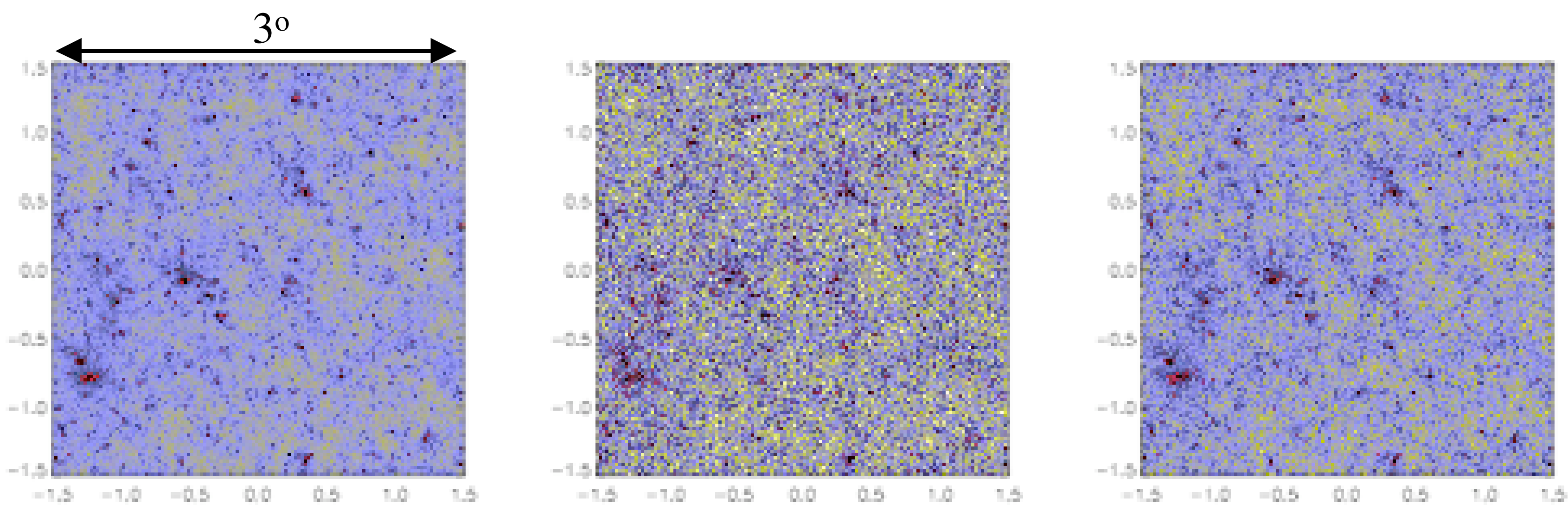
then lensing takes $e \rightarrow e+2\gamma$ (or γ in some conventions). Thus each galaxy provides a (noisy) measure of the shear at its position.

Under the assumption that galaxies are randomly oriented but coherently sheared in some region of the sky, we can simply average the measures of ellipticity to obtain the shear with an error that scales as $e_{rms}/N^{1/2}$ for N galaxies.

Shot noise

For 10% intrinsic ellipticities and 1% shears we need to average over 100 galaxies to get an estimate of the shear at any position on the sky with S/N~1.

Example: simulated convergence maps with appropriate noise



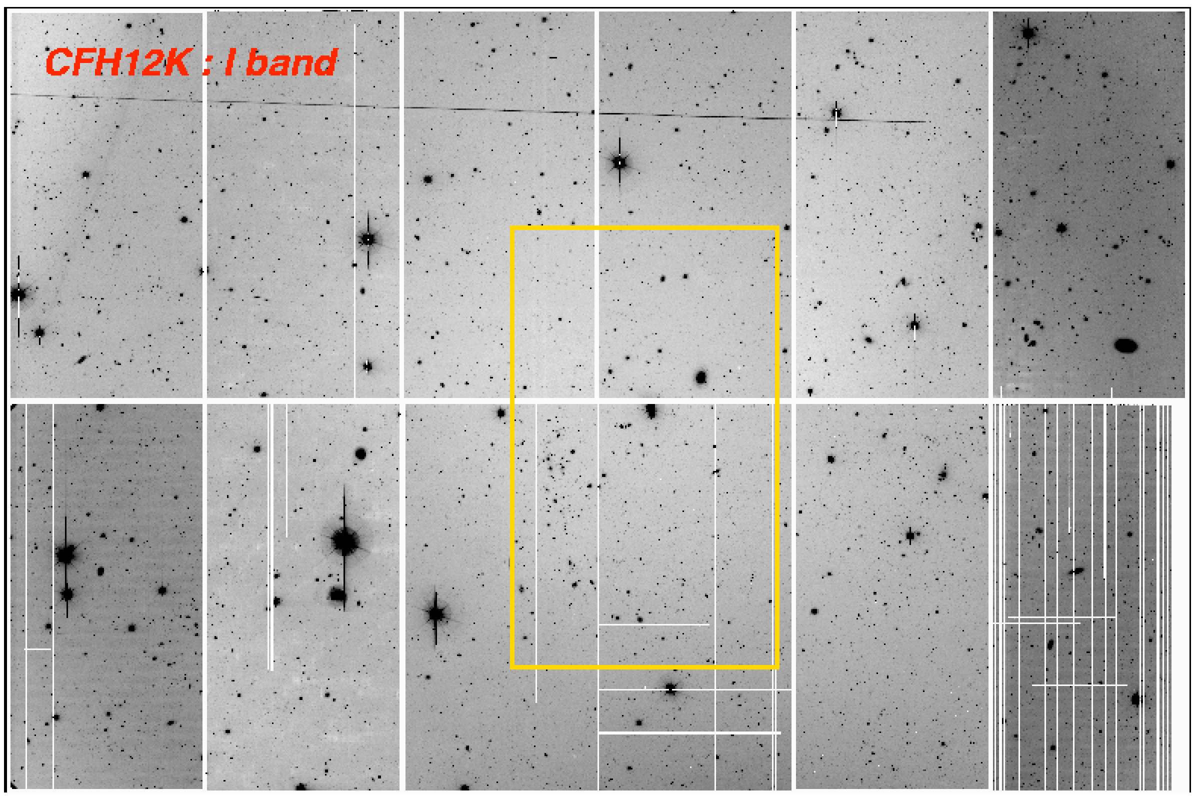
Input

50 gal/arcmin^2

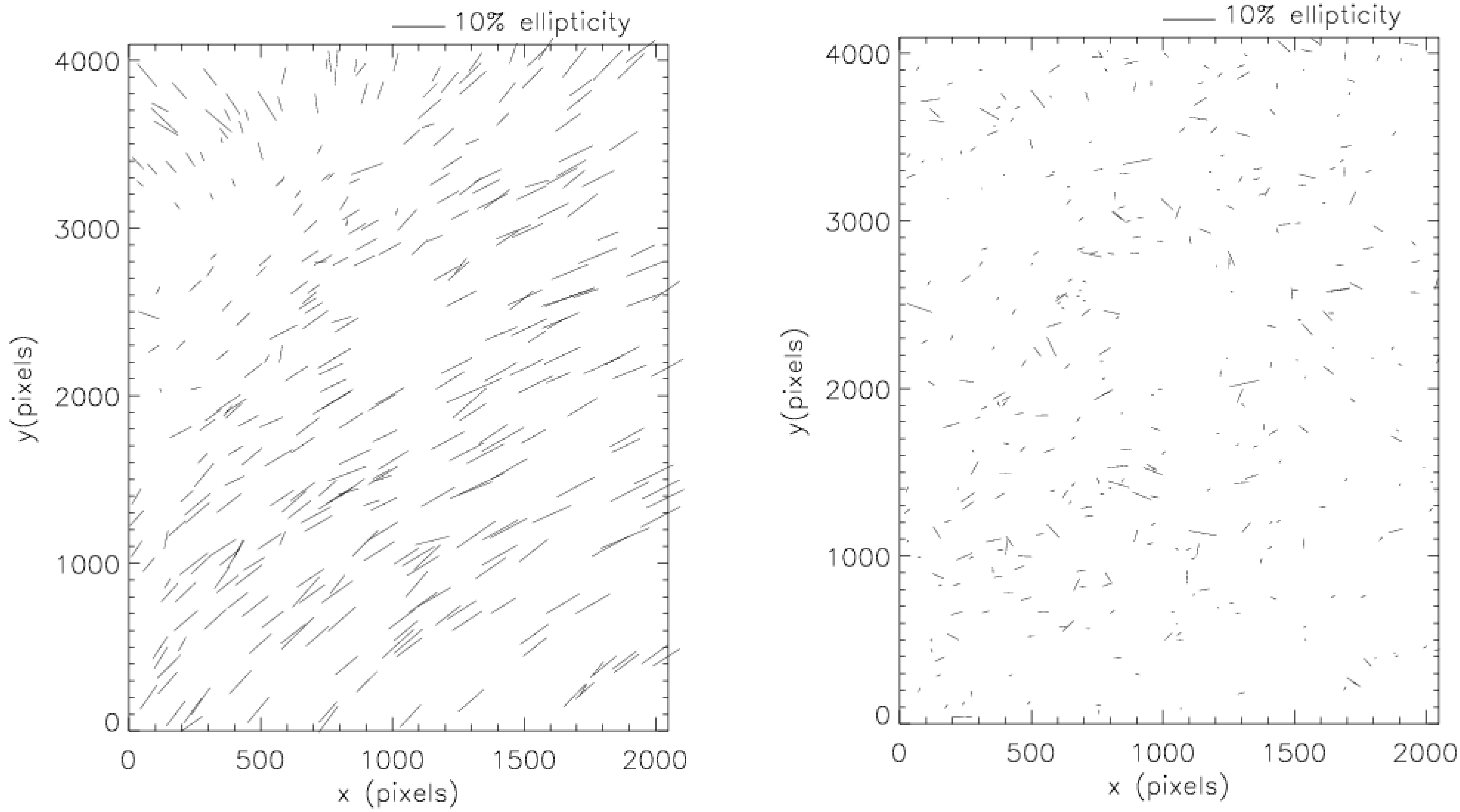
200 gal/arcmin^2

Measuring shear (contd)

Of course, life is not so simple. Observationally one must account for the effects of telescope optics, CCDs, and (often) atmospheric “seeing”.



PSF anisotropy



3-10% rms reduced to ≈0.1%

Correction Method

Without going into details, one corrects the anisotropy by measuring it with stars, modeling it and then removing it from the galaxy images. How this is done, and what assumptions are made, varies from group to group.

- Kaiser, Squires & Broadhurst (1995)
- Bonnet & Mellier (1995)
- Kuijken (1999)
- Kaiser (2000)
- Rhodes, Refregier & Groth (2000)
- Bridle et al. (2001)
- Refregier & Bacon (2001)
- Bernstein & Jarvis (2002)
- Chang & Refregier (2002)
- Hirata & Seljak (2003)

Correction method (details)

Corrections for seeing, PSF etc. all follow a similar derivation:
We take the "true" image and convolve it with some distortion (shear, smear, optics, seeing, …).
How does this affect the measured ellipticities?

$$I'(\theta) = \int d^2\theta' \; I(\theta + \theta')g(\theta')$$

Convolving I(θ) with g implies:

$$M'_{ij} \propto \int d^2\theta \; W(\theta)\theta_i\theta_j I'(\theta) = M_{ij} + q_{\ell m} Z_{\ell m i j}$$

where

$$q_{lm} = \int d^2\theta \; \theta_l \theta_m g(\theta) \qquad Z_{\ell m i j} = \int d^2\theta \; W(\theta)\theta_i\theta_j I_{,\ell m}(\theta)$$

Integrating Z by parts and writing the elements of q_{lm} as q_β we have

$$\delta e_\alpha = P_{\alpha\beta} q_\beta$$

KSB'95

Original method:

PSF Anisotropy: $\varepsilon_g = \varepsilon'_g - P_g P_*^{-1} \varepsilon_*$

PSF Smear & Shear Calibration: $\gamma = (P^\gamma)^{-1} \varepsilon_g$

Intrinsic alignments

We must also worry about intrinsic alignments of galaxies which would violate our “random orientation” hypothesis.

Theoretically we expect such alignments to drop rapidly as the separation of the galaxy pairs is increased which allows us to mitigate the problem observationally (see later).

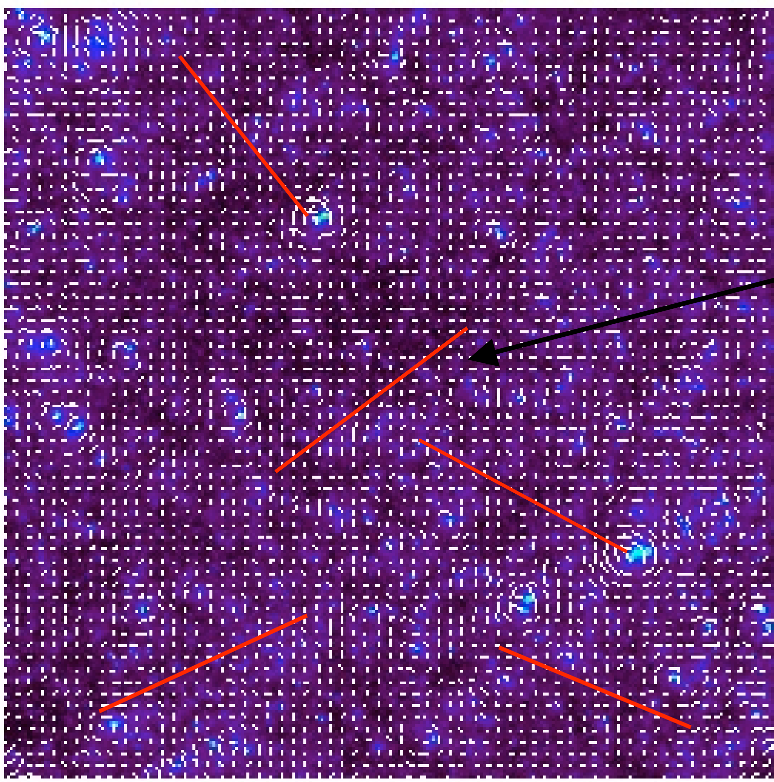
By measuring the “shear” of nearby samples, where lensing is small, we can estimate the size of the intrinsic alignment effect.
It is small!

In principle it is even possible that we will need to worry about correlations in the alignment of a background galaxy with foreground mass which can lens (Hirata & Seljak 2004).

Correlation function(s)

Given a series of measures of "shear" for galaxies *i*, construct estimators of e.g. the correlation function

$$\widehat{\xi}_+(\theta) = \frac{1}{N_{\rm pair}} \sum_{ij} \gamma_+(\vec{\theta}_i)\gamma_+(\vec{\theta}_j)\ \delta\left(\theta - |\vec{\theta}_i - \vec{\theta}_j|\right)$$



Measure γ_1 in the coordinate system aligned with the separation vector.

Transform gives the power spectrum (plus shot noise).

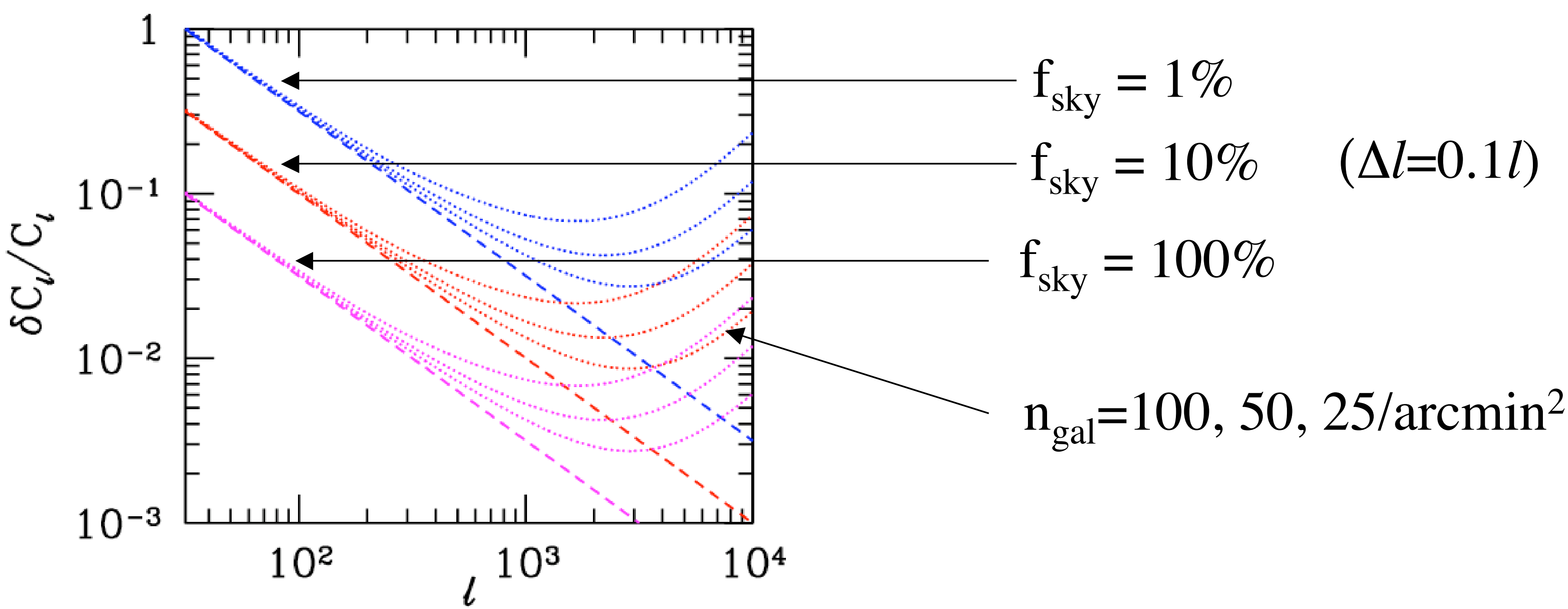
Shear statistics

- Shear variance in cells of size θ
  - Easy to measure
  - Highly correlated
- Power spectra
  - Easy to interpret theoretically
  - Hard to measure with high dynamic range and gappy data.
- Correlation function(s)
  - Handles complex geometries well
  - Correlated errors
- M_{ap} variances on scale θ
  - Produces a scalar from γ field & good E/B separation
  - Mixes scales, and systematics

Measuring the power spectrum

Theorists usally work in terms of the power spectrum C_l.
For a Gaussian field measured over f_{sky} of the sky with a finite number of galaxies the error is:

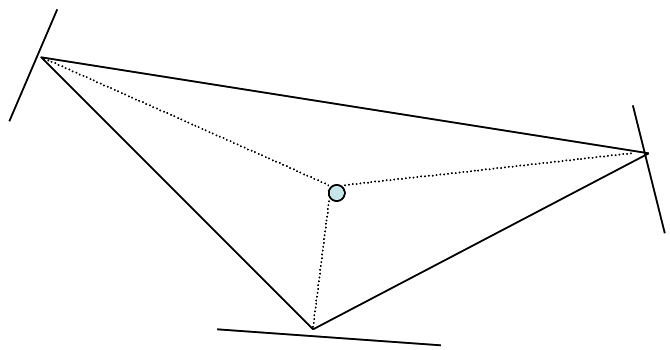
$$\frac{\delta C_\ell}{C_\ell} = \sqrt{\frac{2}{(2\ell+1)f_{\rm sky}}}\left(1+\frac{\gamma^2_{\rm rms}}{\bar{n}_{\rm gal}C_\ell}\right)$$



Higher order statistics

Lensing is clearly non-Gaussian, so there is more to life than the 2-point function. In fact Takada & Jain have shown that there is as much information in the 3-point function as the 2-point fn.

- Shear 3-point correlation function(s)



Line from COM to vertex defines axes for γ_+ and γ_x

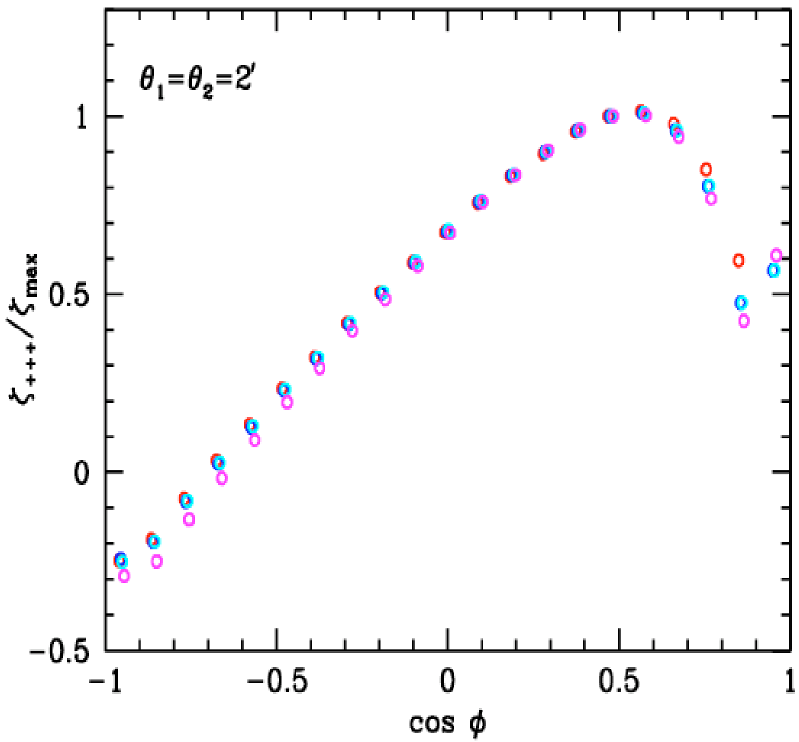
- Higher moments of M_{ap}
- Counts of peaks in M_{ap} or κ from γ field

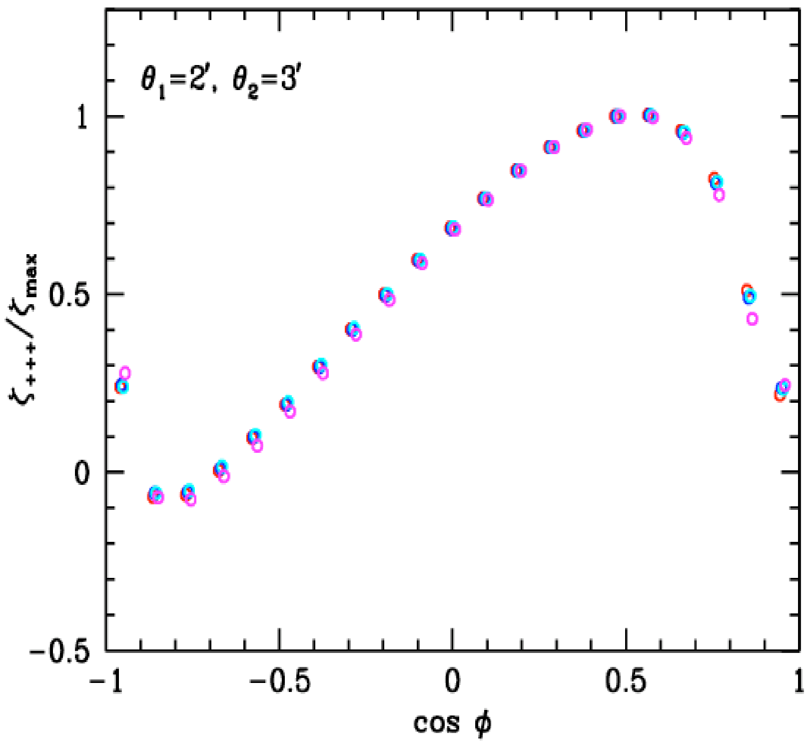
(Best description not yet known …)

The 3PCF

Many functions, many configurations.

Difficult to develop a clean conceptual understanding





Nearly equilateral triangles have the biggest projection of tangential shear onto the +++ component.

The halo model

- "Traditional" methods for treating trans- or non-linear power (e.g. PT, HEPT, etc) don't work very well for lensing. Need a new approach.
- Halo model
  - For estimating the shape of the 3-point function, the errors on cosmological parameters or correlations between C_l bins one can use the "halo model".
  - The calculations can be quite simple: *e.g.* on the scales of relevance the 4-point function, for δC_l, is dominated by the 1-halo term (Cooray et al.).
  - This model borrows heavily from simulations and is more of a "guide" than a precise calculational tool, but is currently sufficient.

Tomography and cosmography

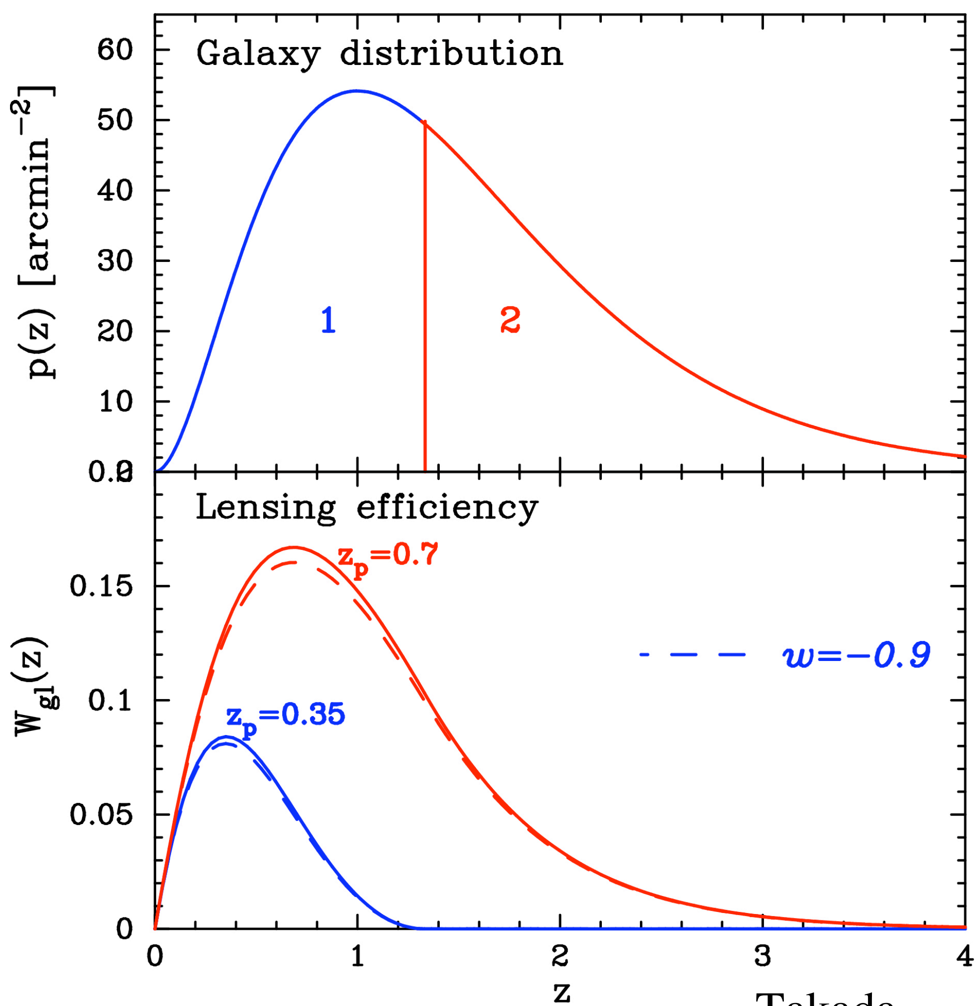
Adding source redshift information

Adding the third dimension

- One of the main limitations of lensing is that it is inherently a projected signal.
- Signal builds up over Gpc along los.
- However if we have source redshift information (spectroscopic or photometric) we can try to break things into slices and regain (some of) the 3^{rd} dimension.

Tomography

- Tomography refers to the use of information from multiple source redshifts.
- This adds some "depth" information to lensing -- important for evolution studies.

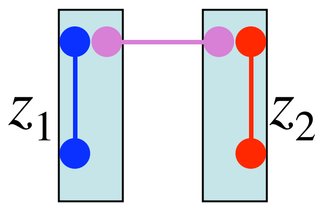


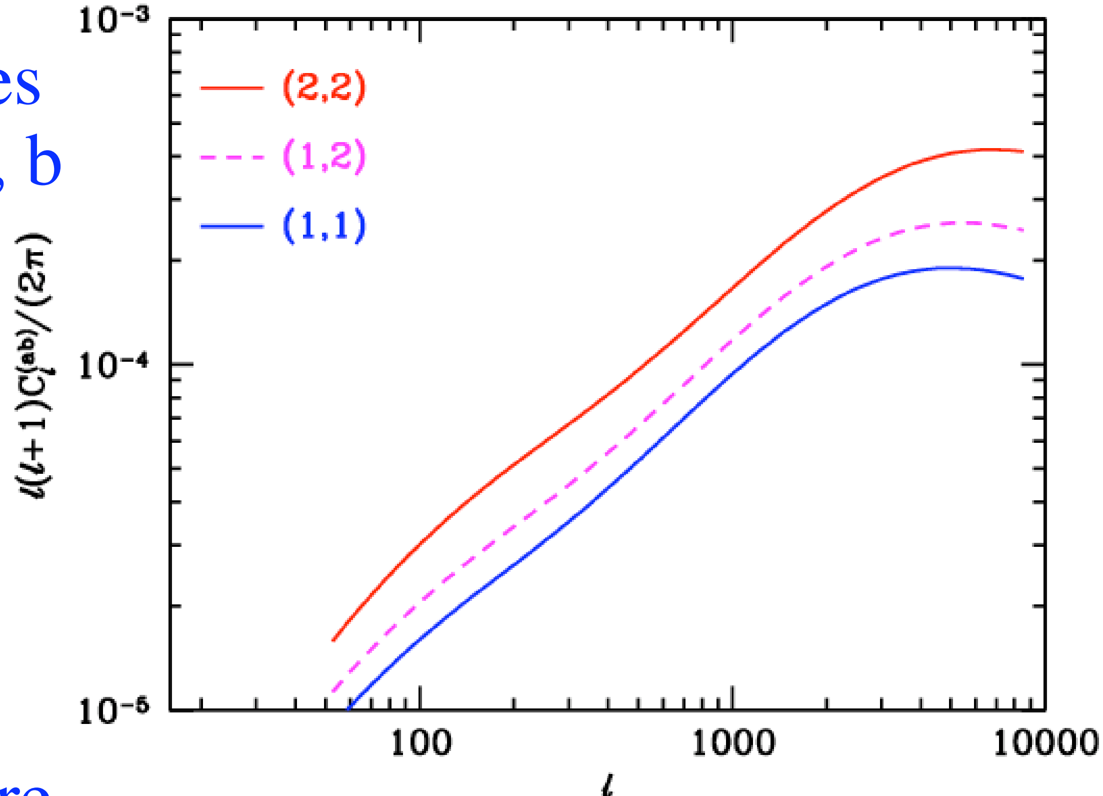
Takada …

Tomography (contd)

The generalization is straightforward for any statistic.

If we divide the sources into bins labelled by a, b then we promote C_l to $C_l^{(ab)}$, etc.





Since $g(\chi)$ is so broad, different source bins are very correlated ($r>0.9$).

Gains saturate quickly!

If ignore $a=b$, then remove almost all intrinsic alignment with little loss of cosmological information for N>3 bins. (Takada & White)

Taylor inversion formula

- As an aside, Andy Taylor has shown that there is an exact inversion of the lensing kernel for the projected potential:

$$\kappa = \frac{1}{2}\partial^2\phi \quad \text{with} \quad \phi = 2\int_0^{\chi} d\chi' \, \left(\frac{\chi - \chi'}{\chi\chi'}\right) \Phi(\hat{n}\chi')$$

- has inverse

$$\Phi(\chi) = \frac{1}{2}\partial_\chi \left[\chi^2 \partial_\chi \phi(\chi)\right]$$

- Practical uses of this formula have not really been found.

C(ross) C(orrelation) C(osmography)

(Jain & Taylor, Bernstein & Jain, Hu & Jain)

- Imagine a single object lensing two sources at z_1 and z_2. For a thin lens of mass Σ_l

$$\kappa_1 = W_{1l}\Sigma_l \quad \text{and} \quad \kappa_2 = W_{2l}\Sigma_l$$

  where the weights depend only on distance ratios for objects of known redshift.
- Taking the ratio of κ's gives a distance ratio of ratios as a function of z, independent of structure!

CCC (contd)

- In principle very clean, but the distance ratio of ratios change only slowly with cosmology, *e.g. w*.
- Need to understand source redshift distribution extremely well:
  - Redshift systematics need to be smaller than 0.1% in ln[1+z]!
- Actual implementation is in terms of cross-correlation of foreground and background shears and galaxies.

Offset-linear scaling

(Zhang, Hui & Stebbins)

$$P_\kappa = \int d\chi_f \, W_f \int d\chi_b \, W_b \int \frac{d\chi}{a^2(\chi)} \left(1 - \frac{\chi}{\chi_b}\right) \left(1 - \frac{\chi}{\chi_f}\right)$$
$$\times \quad P_m(\frac{\ell}{\chi})\Theta(\chi_b - \chi)\Theta(\chi_f - \chi)$$

If W_b and W_f don't overlap can drop first Θ function and the χ_b dependence simplifies to:

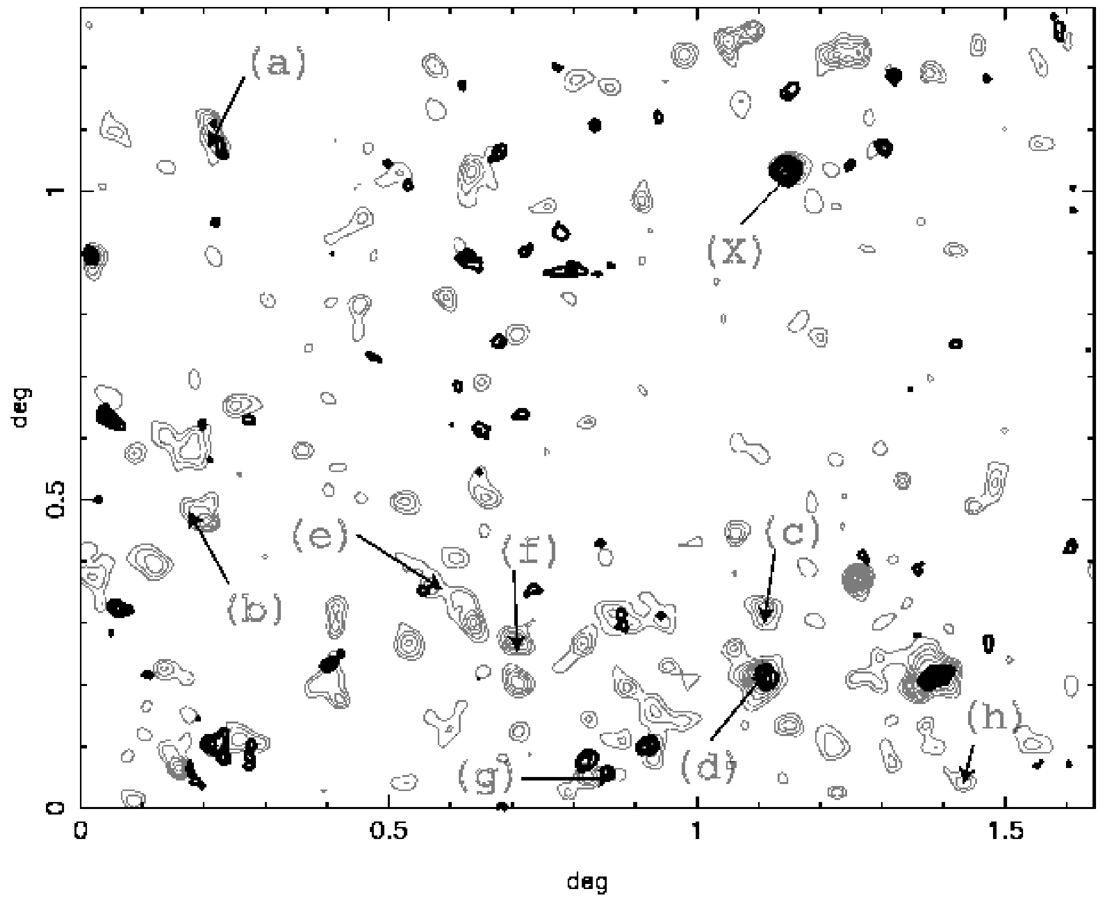
$$P_\kappa = A + B\chi_{\rm eff}^{-1} \quad \text{with} \quad \chi_{\rm eff}^{-1} = \int d\chi_b \, \frac{W_b(\chi_b)}{\chi_b}$$

By measuring P_κ for different z_b can isolate $\chi(z)$!
Like CCC this method is elegant and clean, but not as powerful as using the non-geometric information as well.

Observations

First detections of cosmic shear in Spring 2000

Mass map from 2.1 deg^2 survey with Subaru



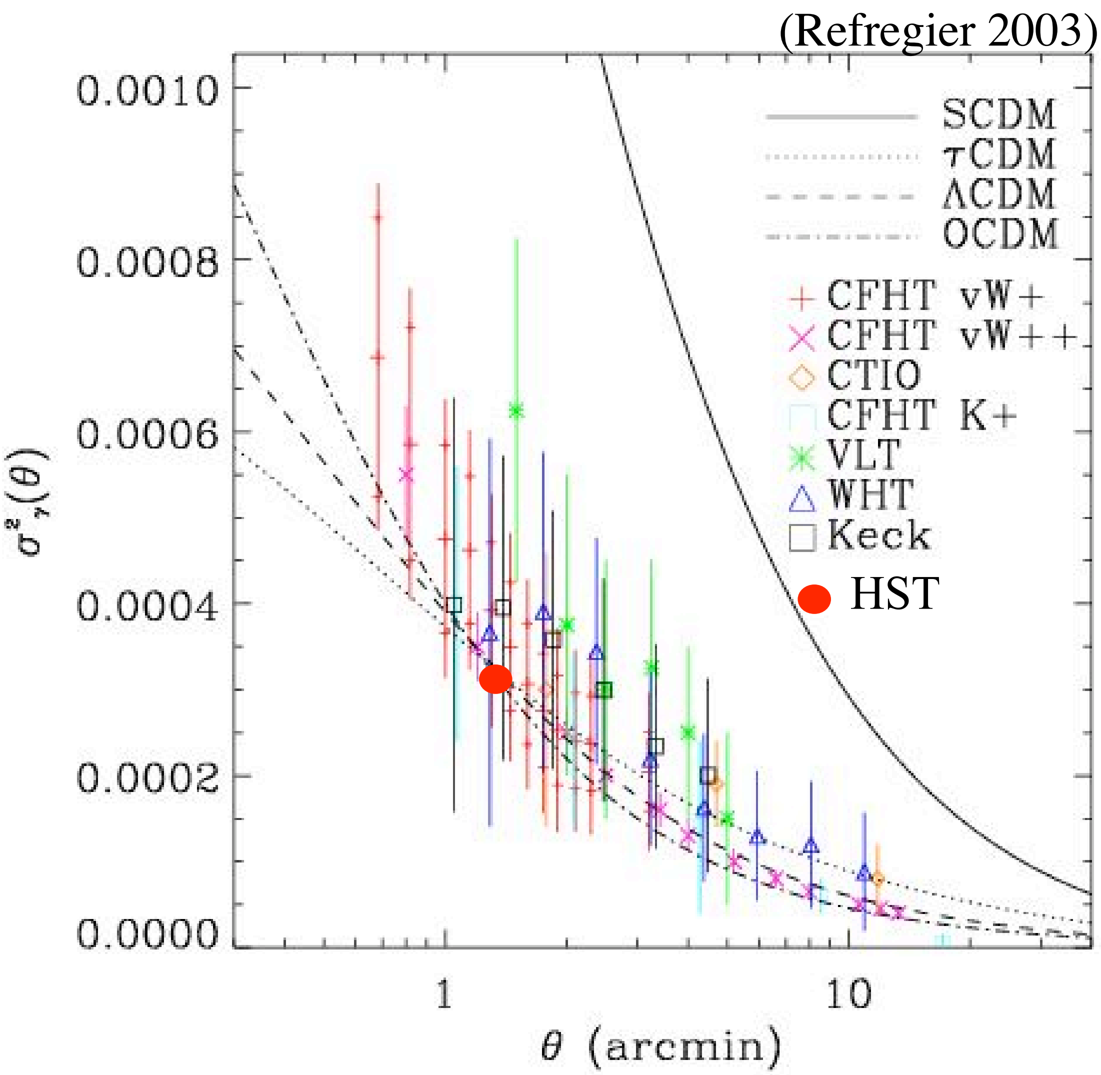
Miyazaki et al. 2002

Observational status through 2003

Typically tens of galaxies per square arcminute

Reference	Year	Telescope	Area (deg^2)	Mag. limit	σ_8
Wittman et al.	2000	CTIO	1.0	$R < 26$	–
van Waerbeke et al.	2000	CFHT	1.7	–	–
Kaiser et al.	2000	CFHT	1.0	$I < 24$	–
Bacon et al.	2000	WHT	0.5	$R < 26$	$1.50^{+0.50}_{-0.50}$
Maoli et al.	2001	VLT	0.7	$I < 25$	$1.03^{+0.03}_{-0.03}$
Rhodes et al.	2001	HST	0.05	$I < 26$	$0.91^{+0.21}_{-0.30}$
van Waerbeke et al.	2001	CFHT	6.5	$I < 25$	$0.88^{+0.02}_{-0.02}$
Hammerle et al.	2002	HST	0.02	–	–
Hoekstra et al.	2002	CFHT	24	$R < 24$	$0.81^{+0.07}_{-0.09}$
van Waerbeke et al.	2002	CFHT	8.5	$I < 25$	$0.98^{+0.06}_{-0.06}$
Refregier et al.	2002	HST	0.4	$I < 24$	$0.94^{+0.14}_{-0.14}$
Bacon et al.	2002	WHT	1.6	$R < 26$	$0.97^{+0.13}_{-0.13}$
Hoekstra et al.	2002	CFHT	53	$R < 24$	$0.86^{+0.04}_{-0.05}$
Jarvis et al.	2002	CTIO	75	$R < 23$	$0.71^{+0.06}_{-0.08}$
Brown et al.	2003	ESO	1.3	$R < 25$	$0.72^{+0.09}_{-0.09}$
Hamana et al.	2003	Subaru	2.1	$R < 26$	$0.69^{+0.18}_{-0.13}$

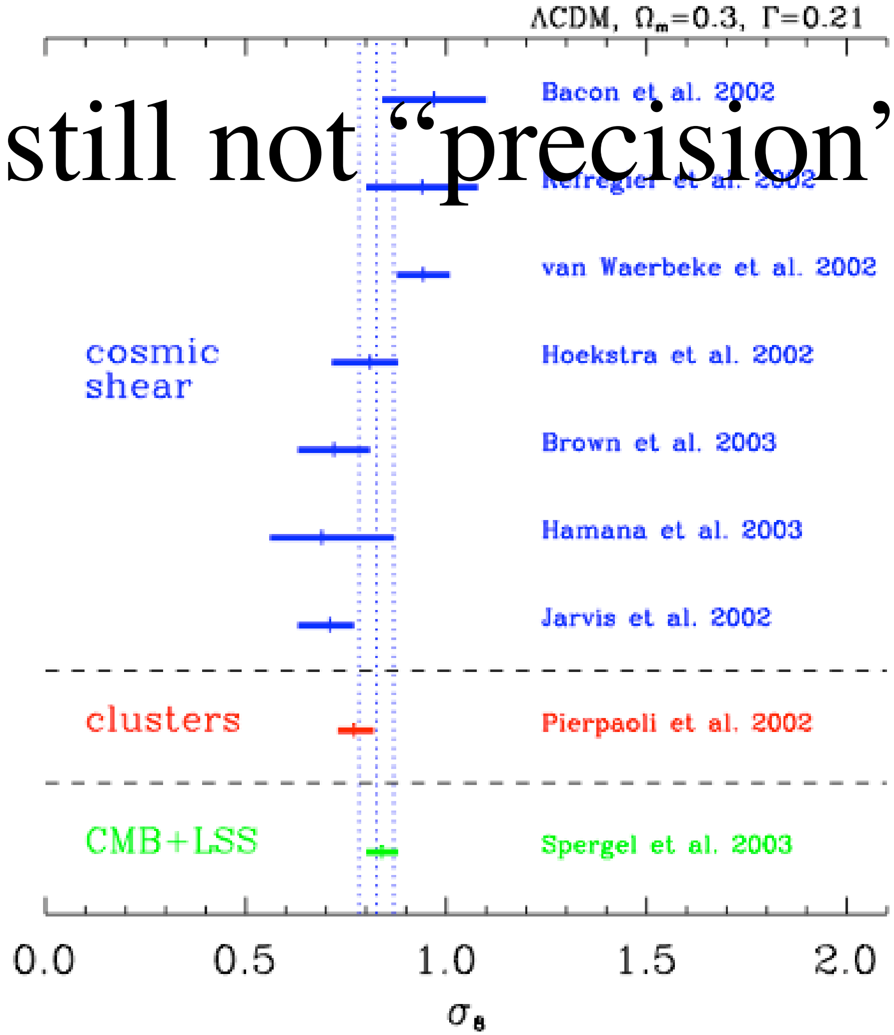
Agreement isn't bad



Different measurements are roughly consistent, though they have different source *z*-distributions etc.

In agreement with cluster-normalized CDM model

But still not "precision" cosmology

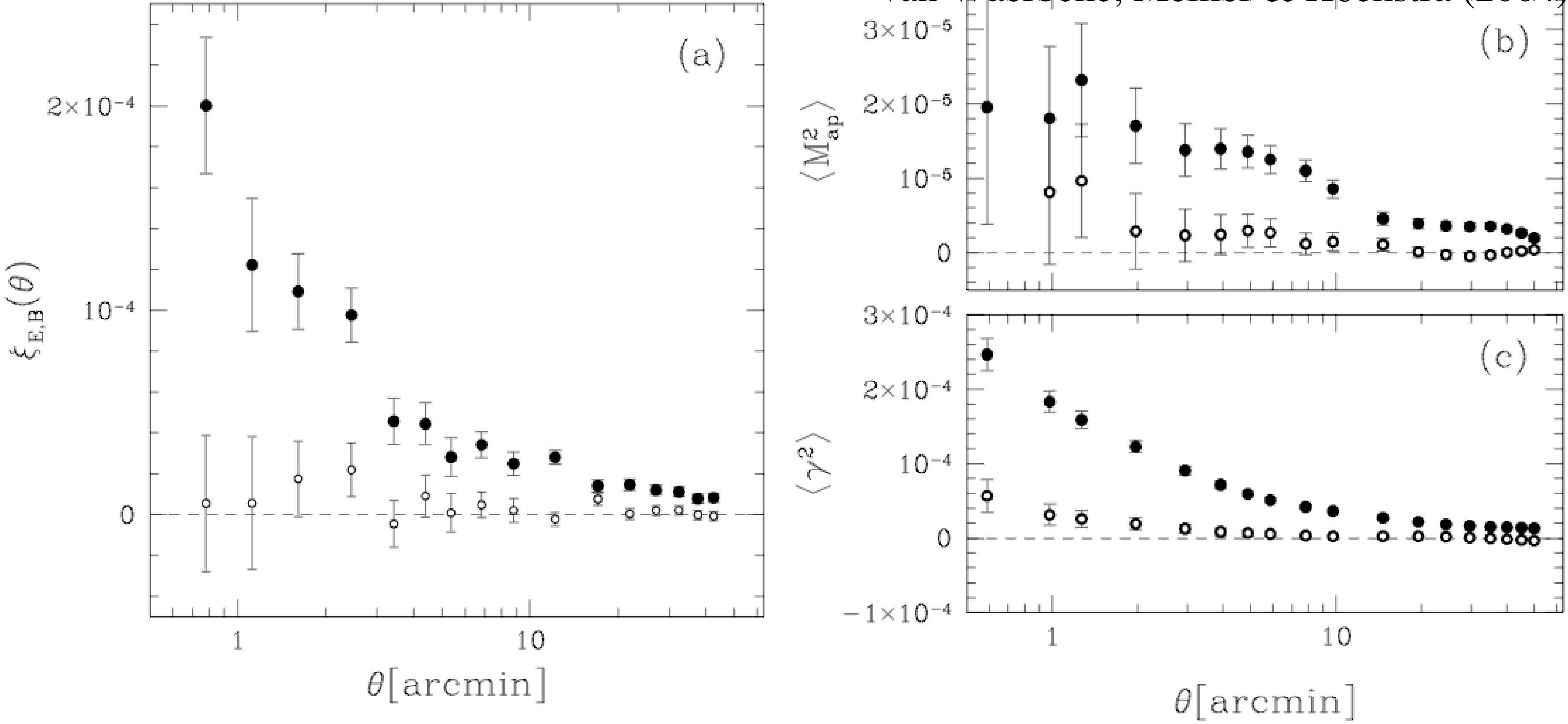


Need the equivalent of 1% precision in σ_8 to be able to measure dark energy *w*.

Refregier (ARAA, 2003)

The 2-point function: state of the art

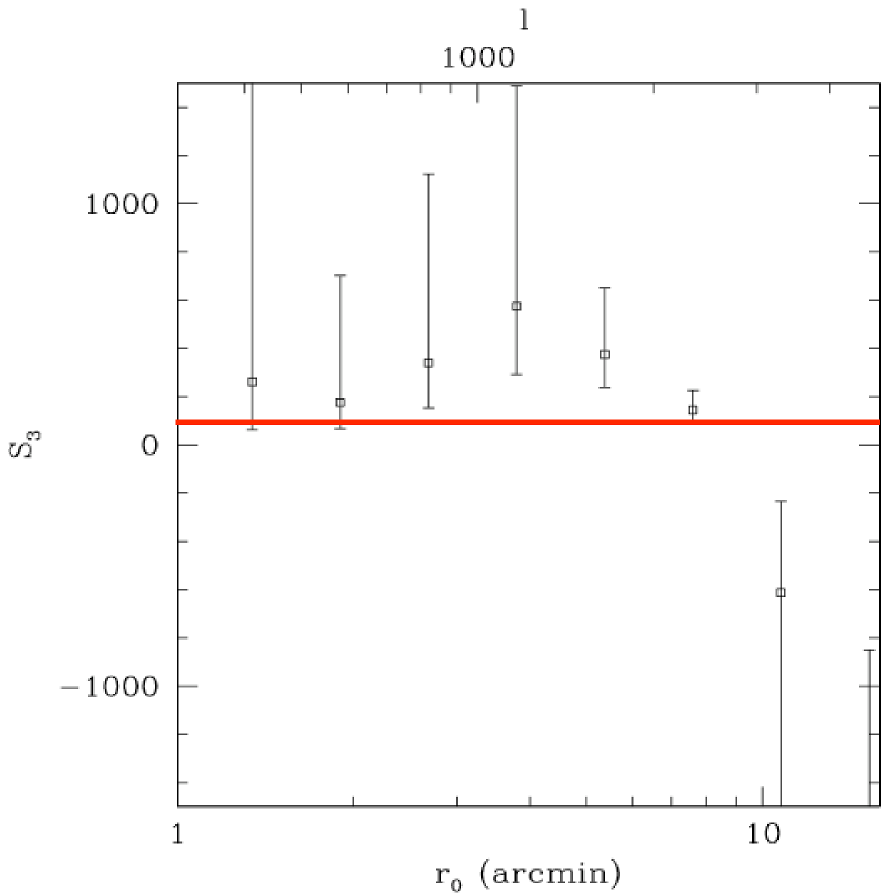
van Waerbeke, Mellier & Hoekstra (2004)



We are beginning to measure the power spectrum. B-modes gone!

The skewness

By measuring the 2- and 3-point functions of the shear, the VIRMOS-DESCARTES group (Pen et al. 2003) were able to compute $S_3 = \langle\kappa^3\rangle / \langle\kappa^2\rangle^2$ over a range of scales.



The errors include an allowance for the non-zero *B*-mode they found during this earlier analysis.
Re-analysis in progress.

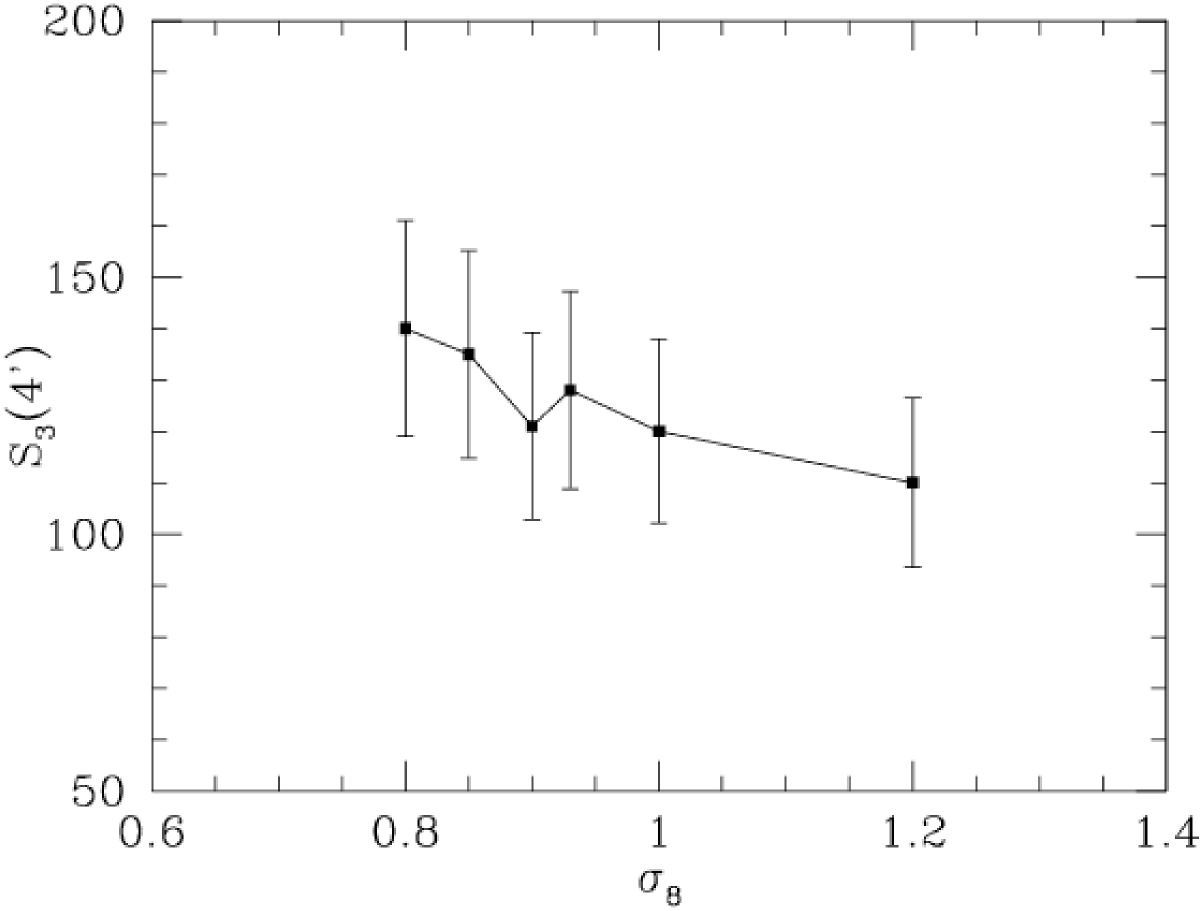
Skewness in theory

There is still some disagreement about the predicted value of S_3 (and S_4) from simulations/theory.
Value fluctuates from box to box. Error is ~10-20%.

	σ_8	$S_3(1')$	$S_3(4')$	$S_4(1')$	$S_4(4')$
VWb	0.8	155	140	5.8	4.6
WVb	0.85	145	135	5.0	4.1
JSW	0.9	145	140	3.8	3.1
HM	0.9	118	110	2.9	2.6
H2	0.9	138	114	4.5	3.4
WVa	0.93	133	128	3.9	3.8
VWa	1.0	138	120	4.1	2.8
WH	1.2	-	110	-	2.8
v.W	0.9	140	127	-	-
TJ	0.9	137	147	3.5	4.2

(units of 10^4)

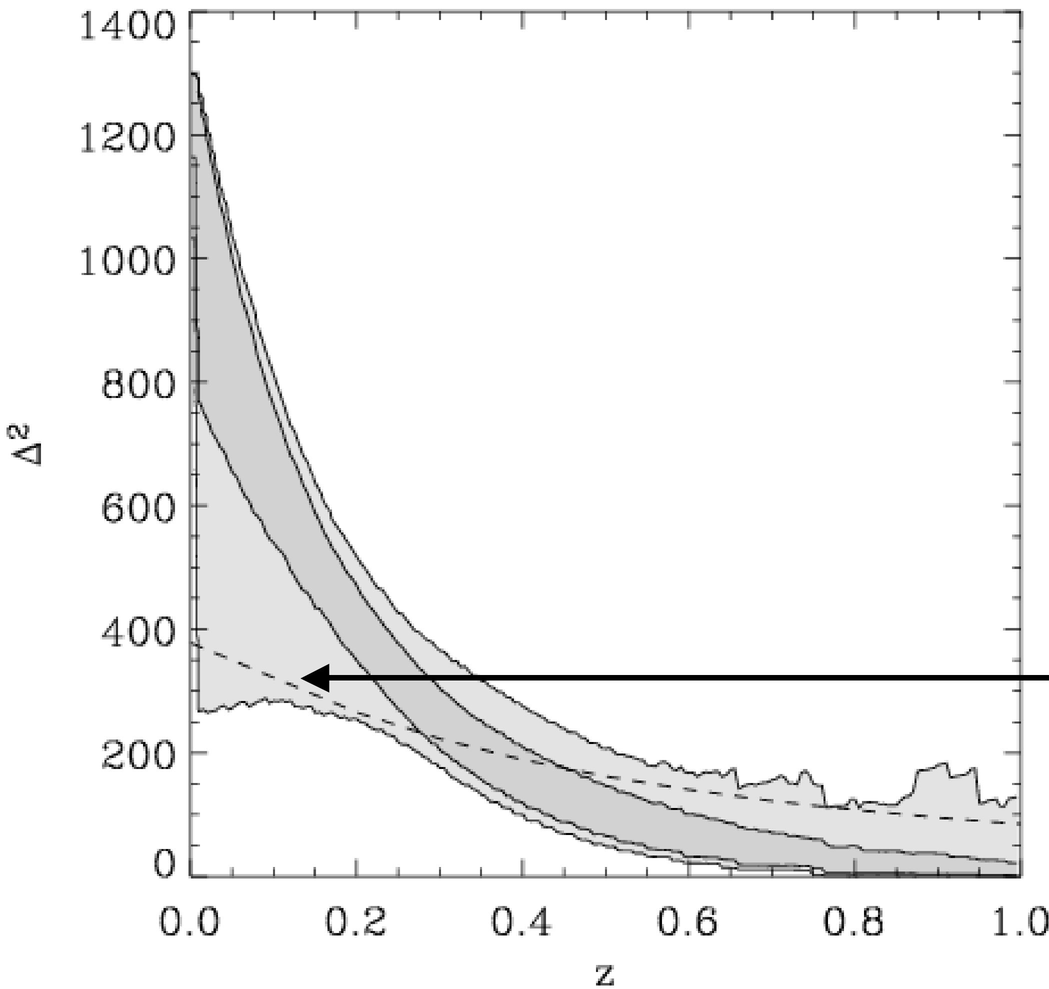
Skewness vs clustering amplitude



Errors are an estimate of run-to-run scatter.

PT predicts an S_3 independent of σ_8.

Structure grows!?



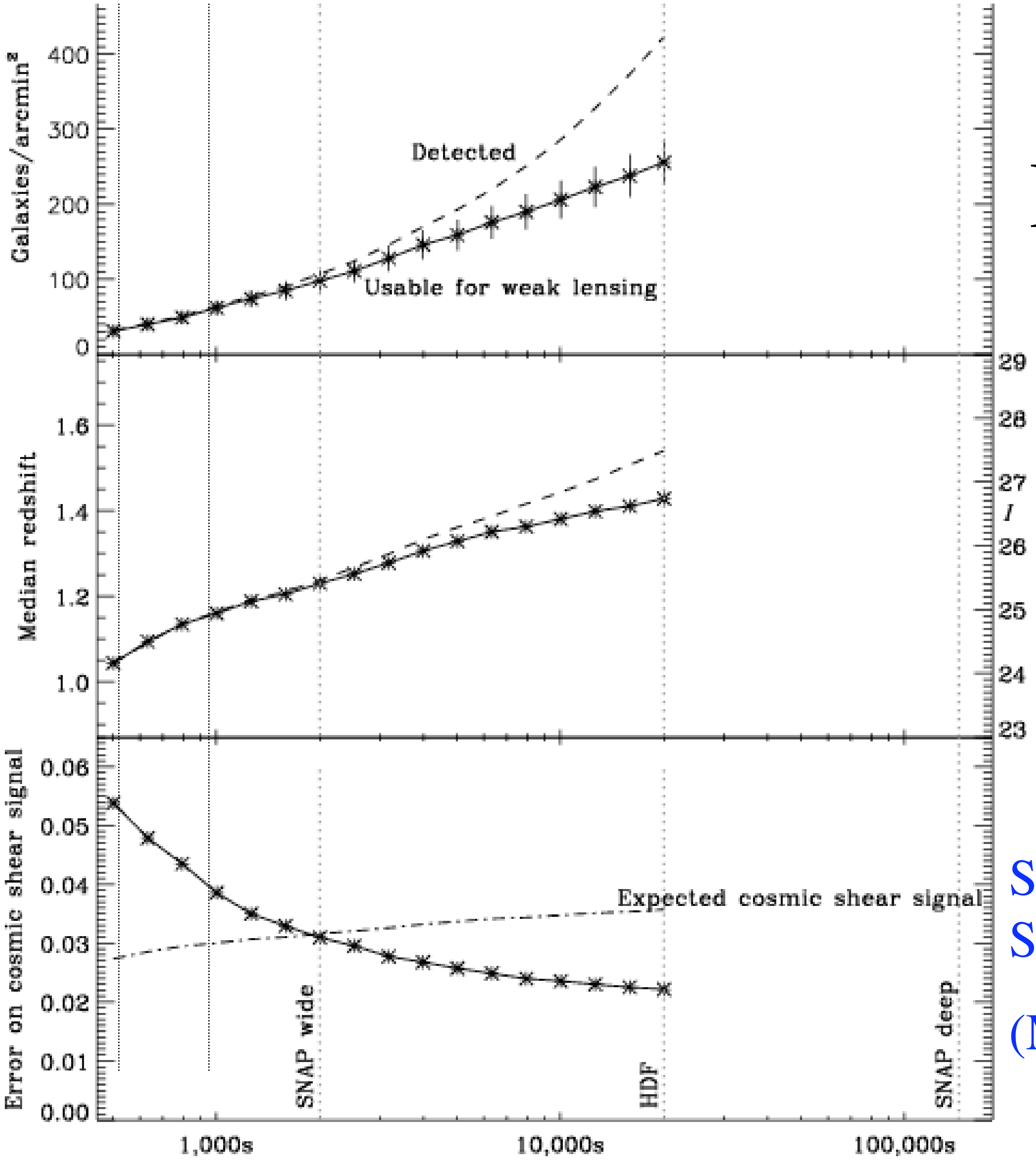
Evolution of power, Δ^2(14/Mpc), from the COMBO-17 survey.

(Bacon *et al*. 2004)

ΛCDM with σ_8=0.7

Future projects

Survey	Diameter (m)	FOV (deg^2)	Area (deg^2)	Start
DLS	2×4	2×0.3	28	1999
CFHT-LS	3.6	1	172	2003
VST	2.6	1	x100	2004
VISTA	4	2	10000	2007
Pan-STARRS	4×1.8	4×4	31000	2008
DES	4	2.1	5000	200X
LSST	8.4	7	30000	201X
SNAP	2	0.7	300-7000	201X



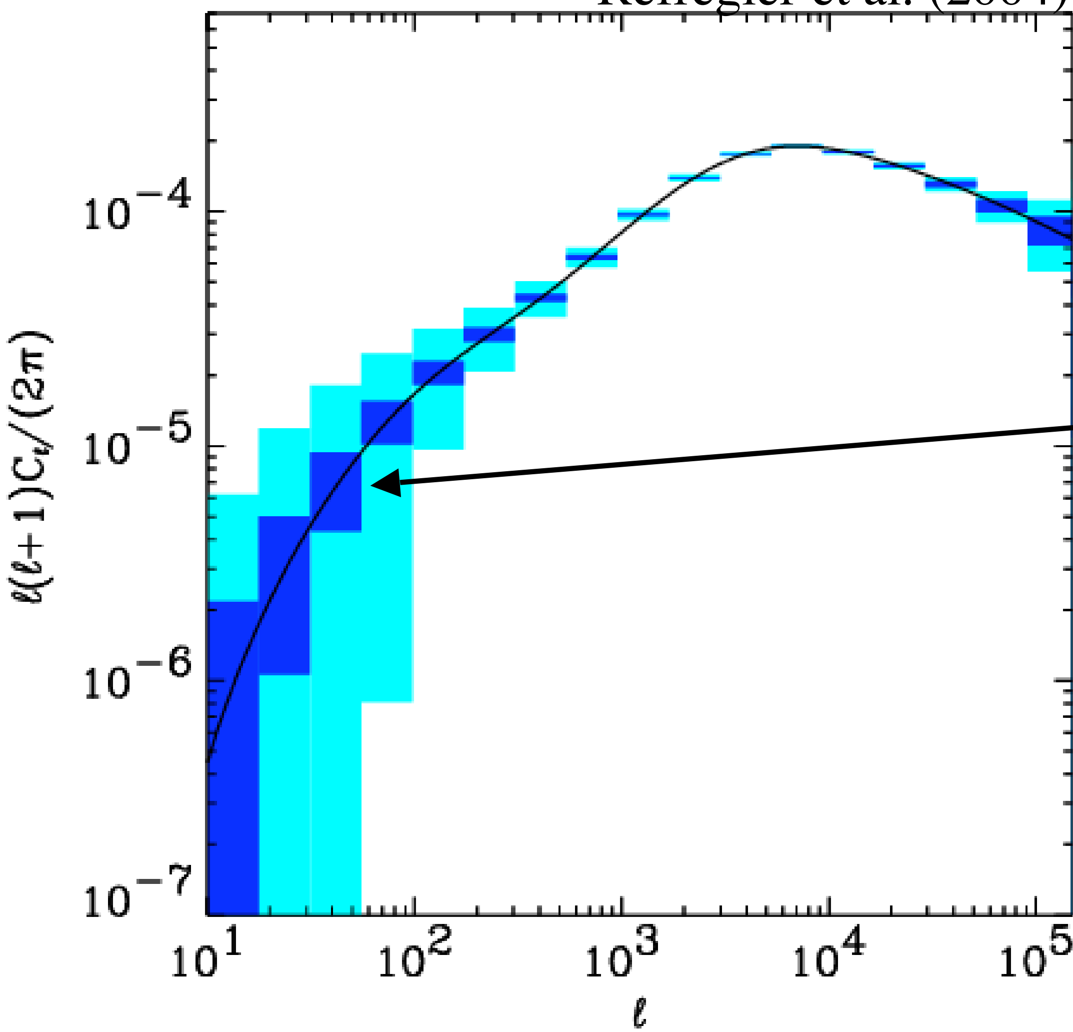
How many galaxies?

Simulations from the SNAP WLWG.

(Massey *et al*. 2004)

The power spectrum with SNAP

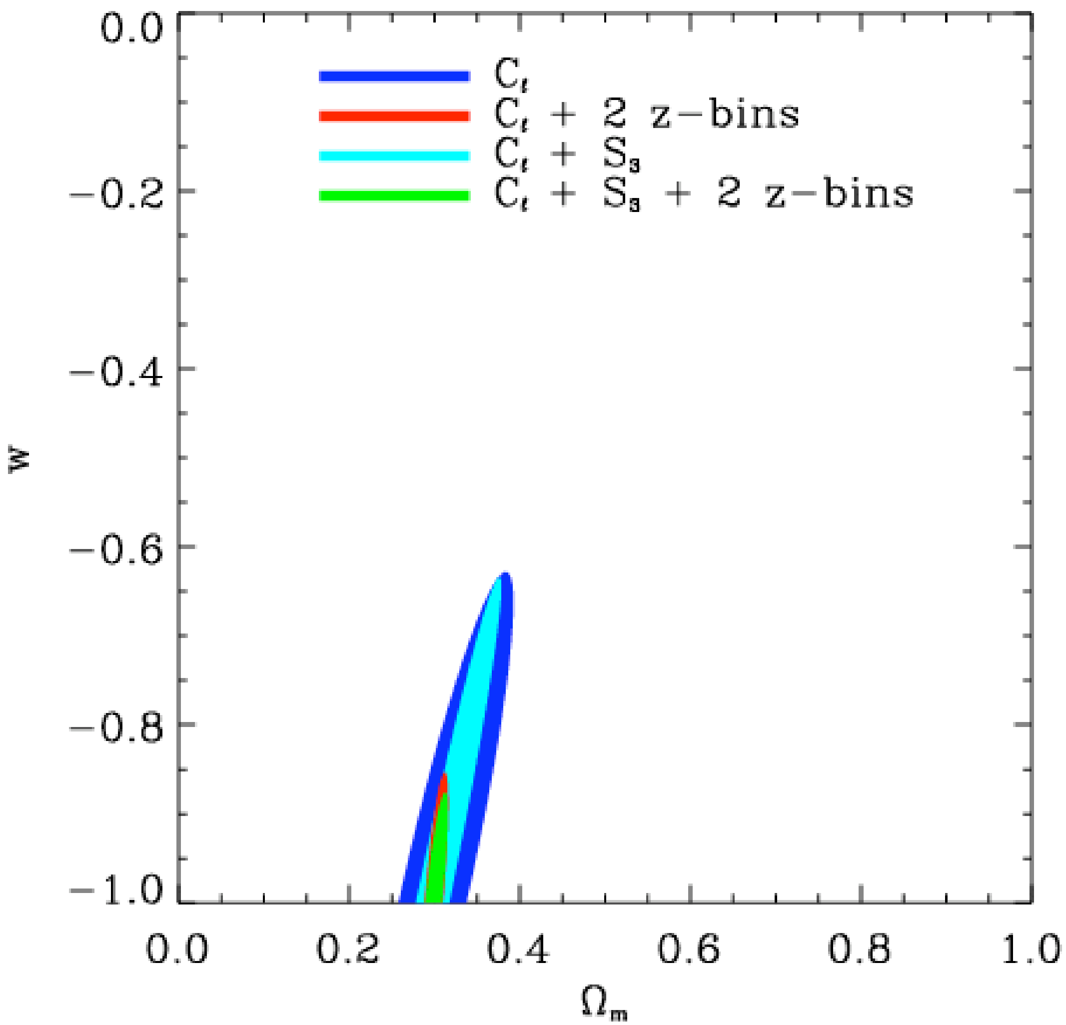
Refregier et al. (2004)



A “minimal” survey with100 gal/arcmin^2 over 300 sq. deg.

Errors scale as $f_{sky}^{-1/2}$

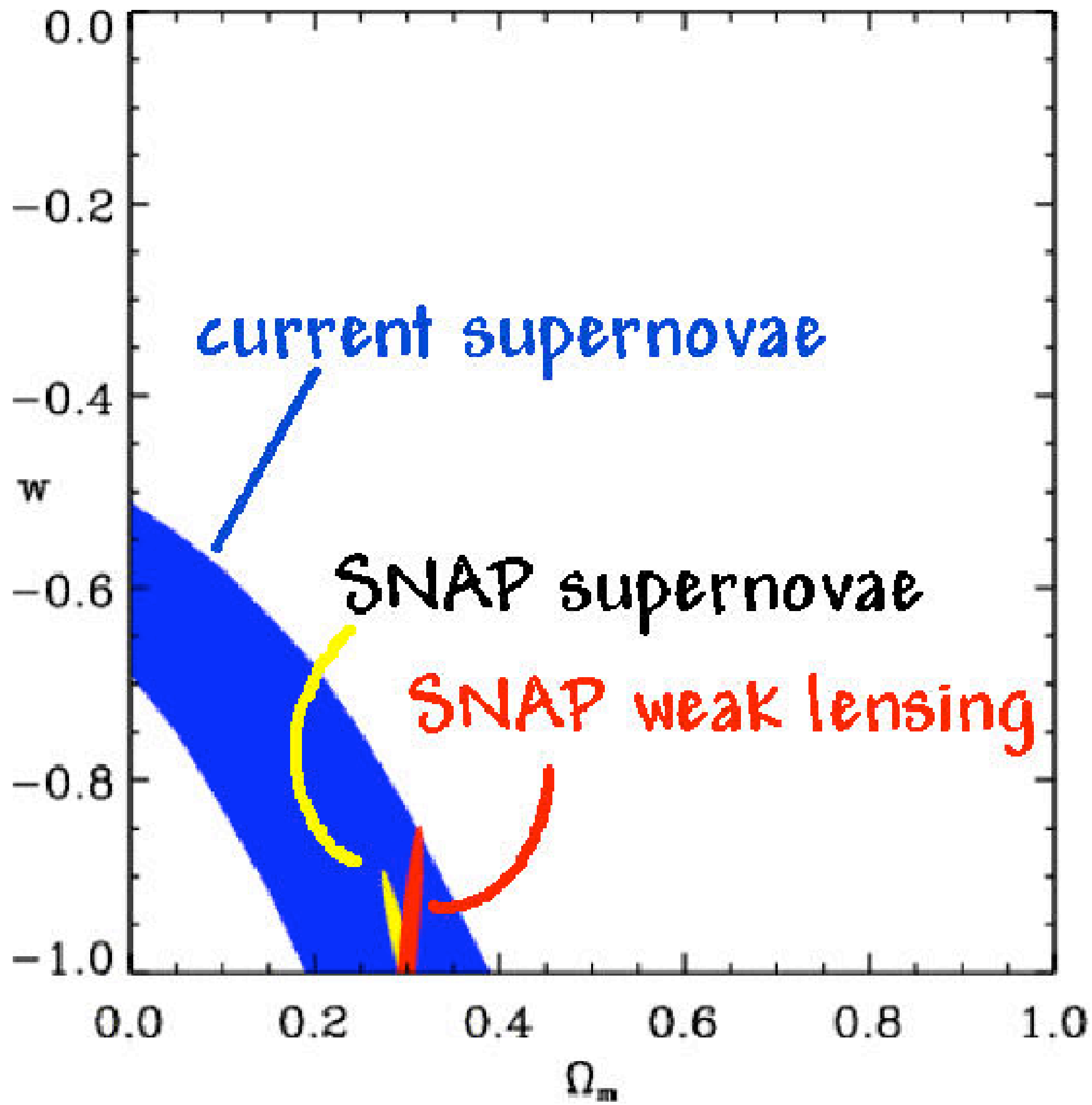
Tomography and skewness



Constraints on DE are improved by using tomography and adding non-Gaussian information.

(from Refregier et al.)

Lensing and dark energy

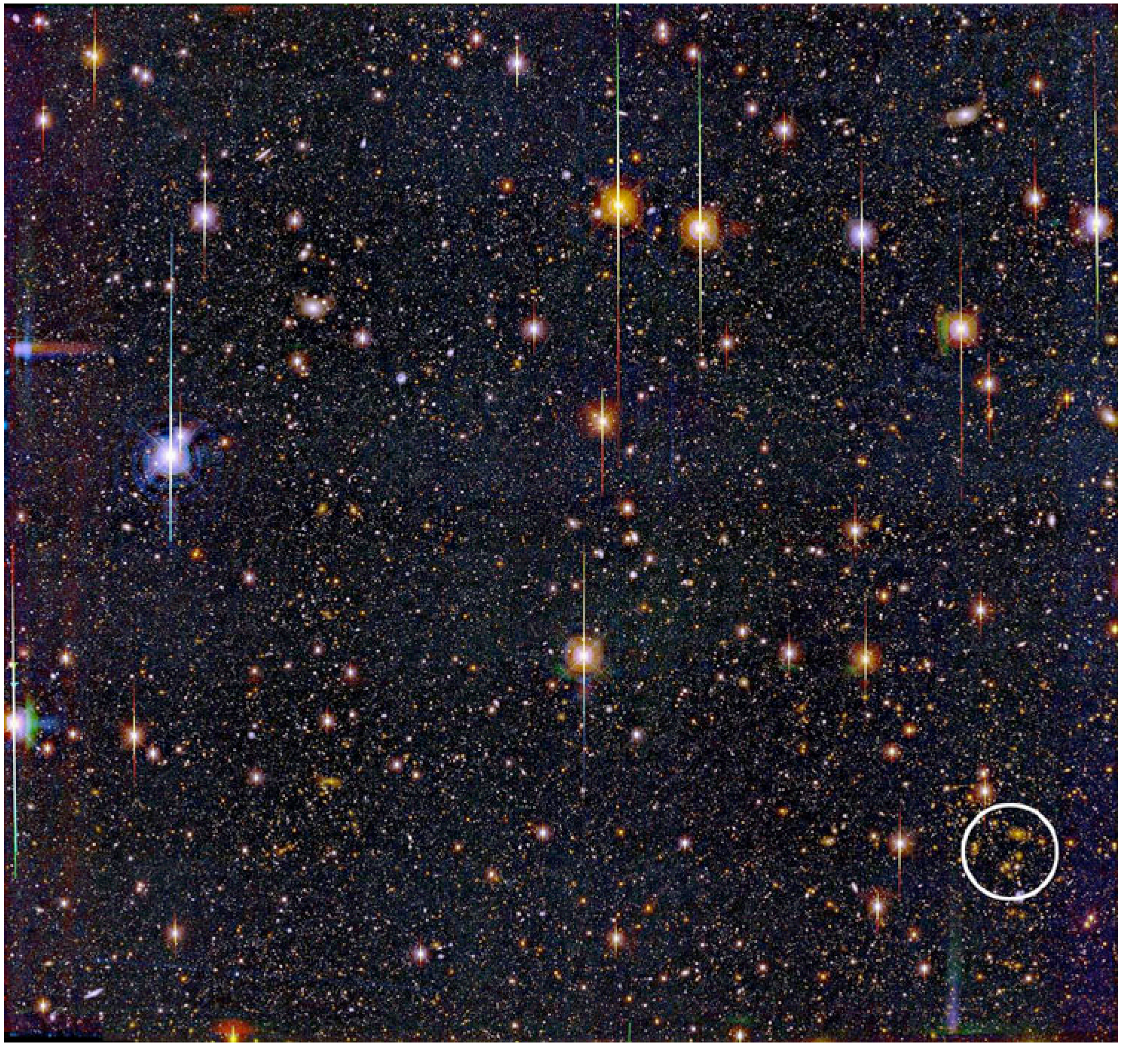


Finding clusters with weak lensing

- The obvious extension of non-Gaussian thinking is to look at the extrema of the maps -- finding clusters.
- Lensing depends only on the mass distribution (and the cosmology)
- Massive structures generate detectable shear
- Thus lensing is a good way to find clusters, but
- Lensing measures the *projected* mass along the line of sight.

First Shear-Selected Cluster

(Wittman et al. 2001)

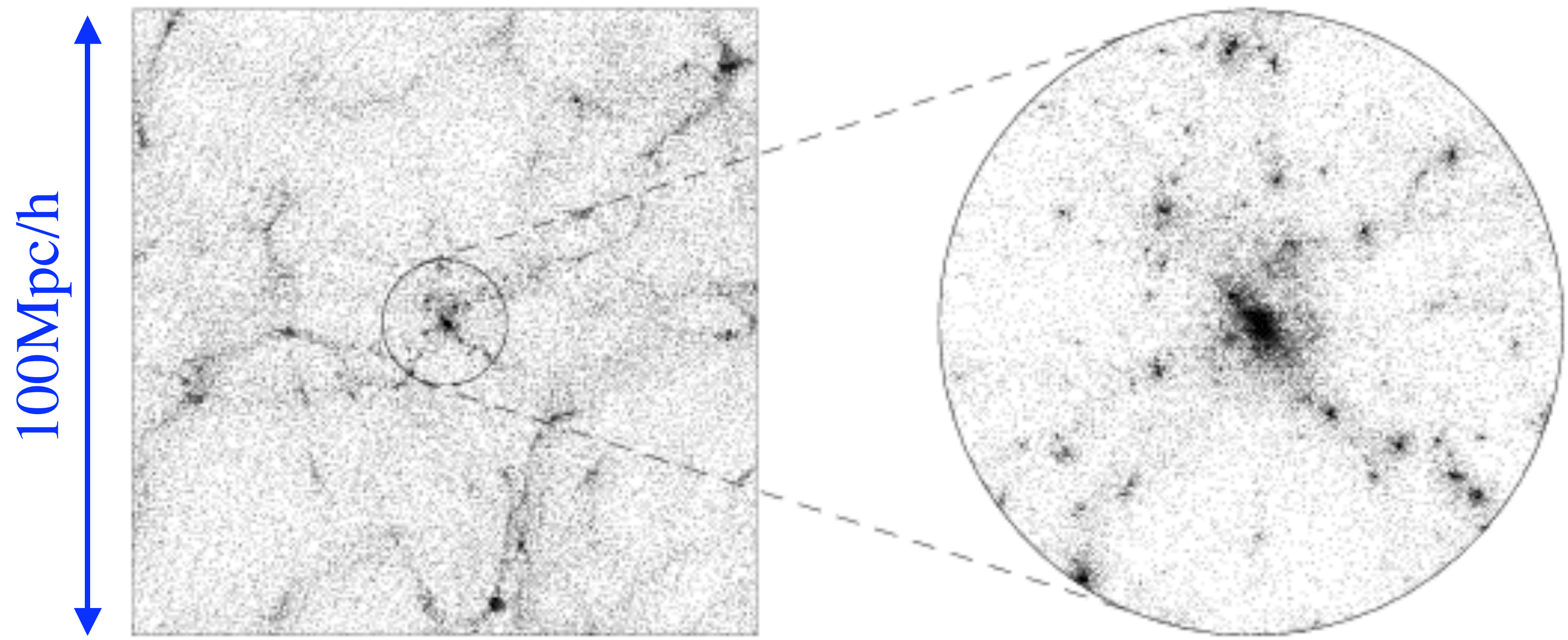


40’

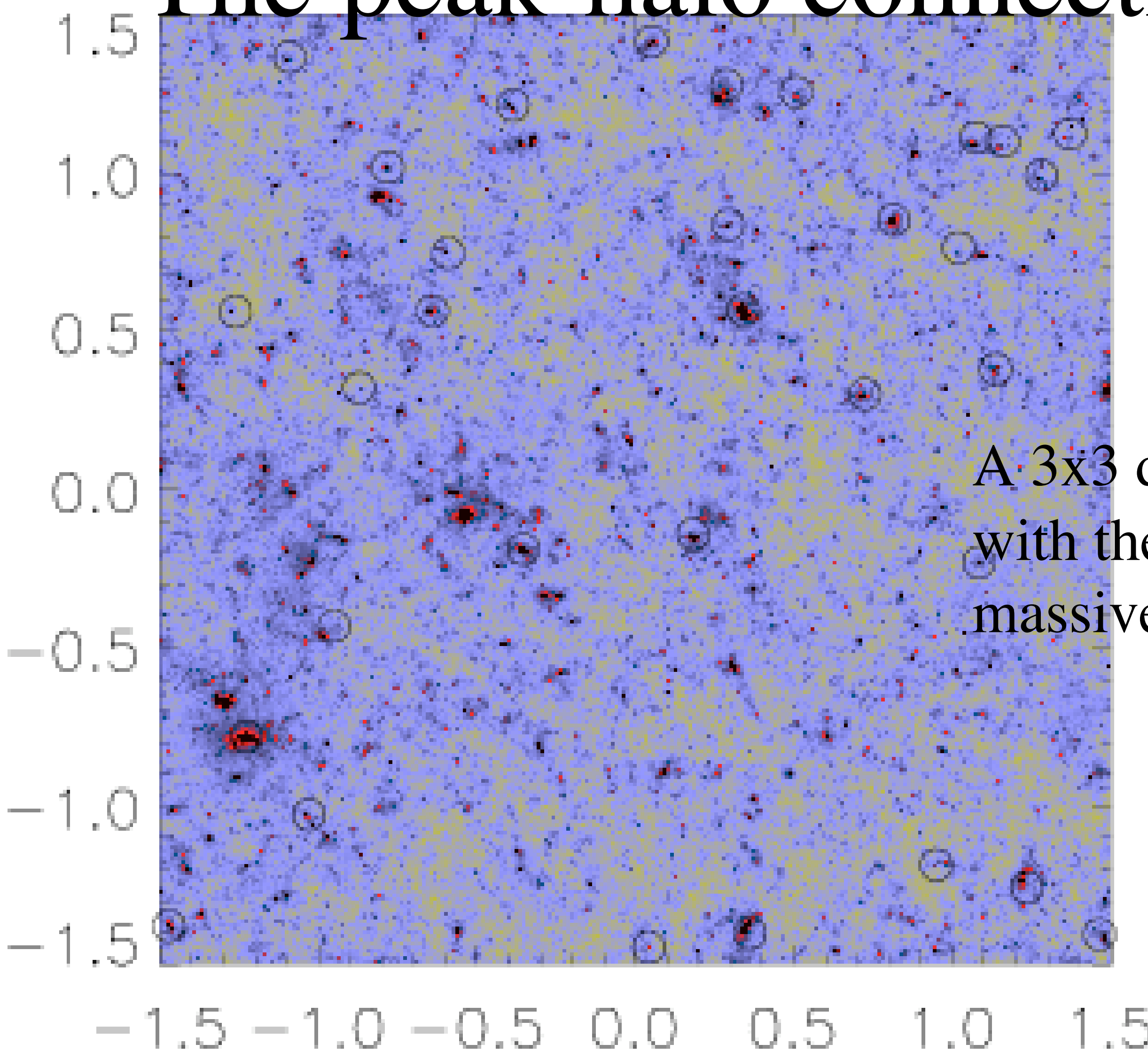
- First cluster discovered through its lensing effect rather than radiation!
- σ_{vel} = 615 km s^{-1}
- $z_{spectra}$ = 0.28

Projection effects lead to scatter in the shear-mass relation

Scatter in the shear-mass relation means lensing does *not* produce a mass selected sample, but a shear selected sample! This has implications for doing cosmology.



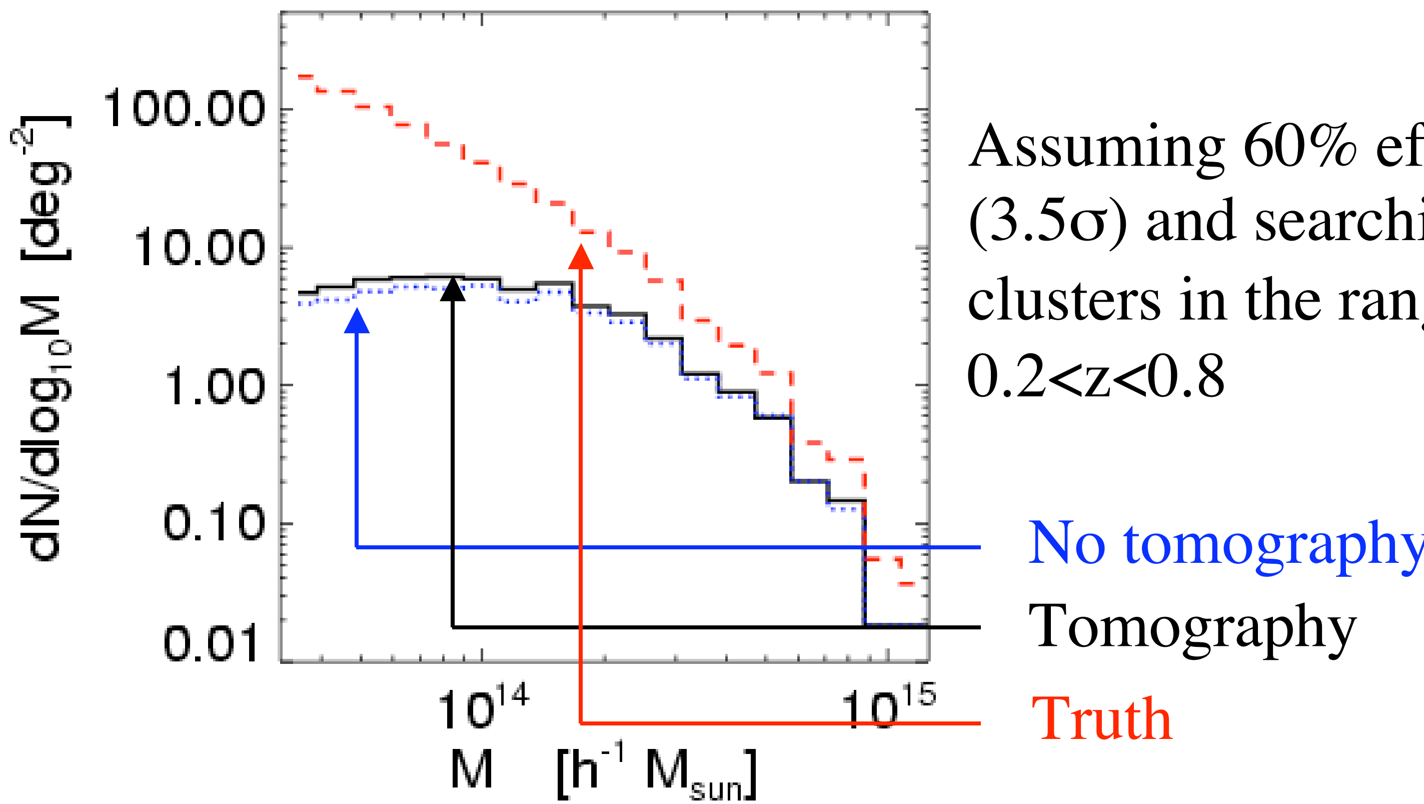
The peak-halo connection



A 3x3 degree κ map with the 32 most massive halos circled!

Projection effects can be severe

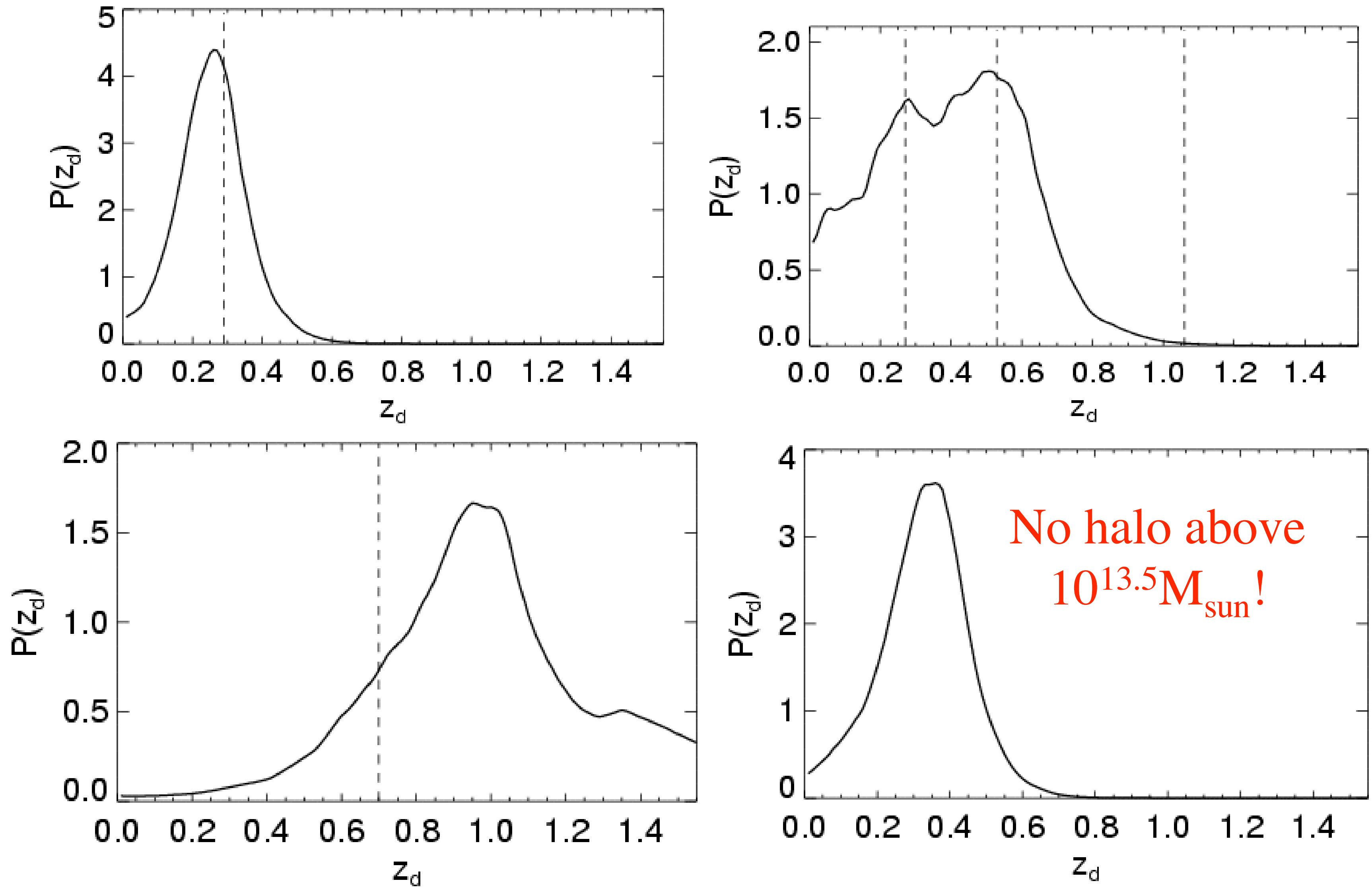
Hennawi & Spergel have used a tomographic matched filter algorithm to find clusters and determine their redshifts. The tomographic information helps reduce projection effects, but cannot eliminate them entirely.



Assuming 60% efficiency (3.5σ) and searching for clusters in the range 0.2<z<0.8

Tomographic (MF) redshifts

(Hennawi and Spergel 2004)



Simulations

Types and uses of simulations

Lensing lends itself to numerical simulation …

We need numerical simulations to refine and calibrate algorithms and analytic approximations, and potentially serve as templates when the data become available.

Simulations can be used to extract:

- Halo abundances and shapes
- Mass power spectra (and covariance matrices)
- Projected mass maps
- Ray tracing maps
- Mock galaxy catalogues

The MLP algorithm

- The gold standard of simulation algorithms is the "multiple lens plane" algorithm, where we trace ray bundles through the evolving mass distribution in an N-body simulation.
- The lensing equations are discretized and the integrals turned into sums:

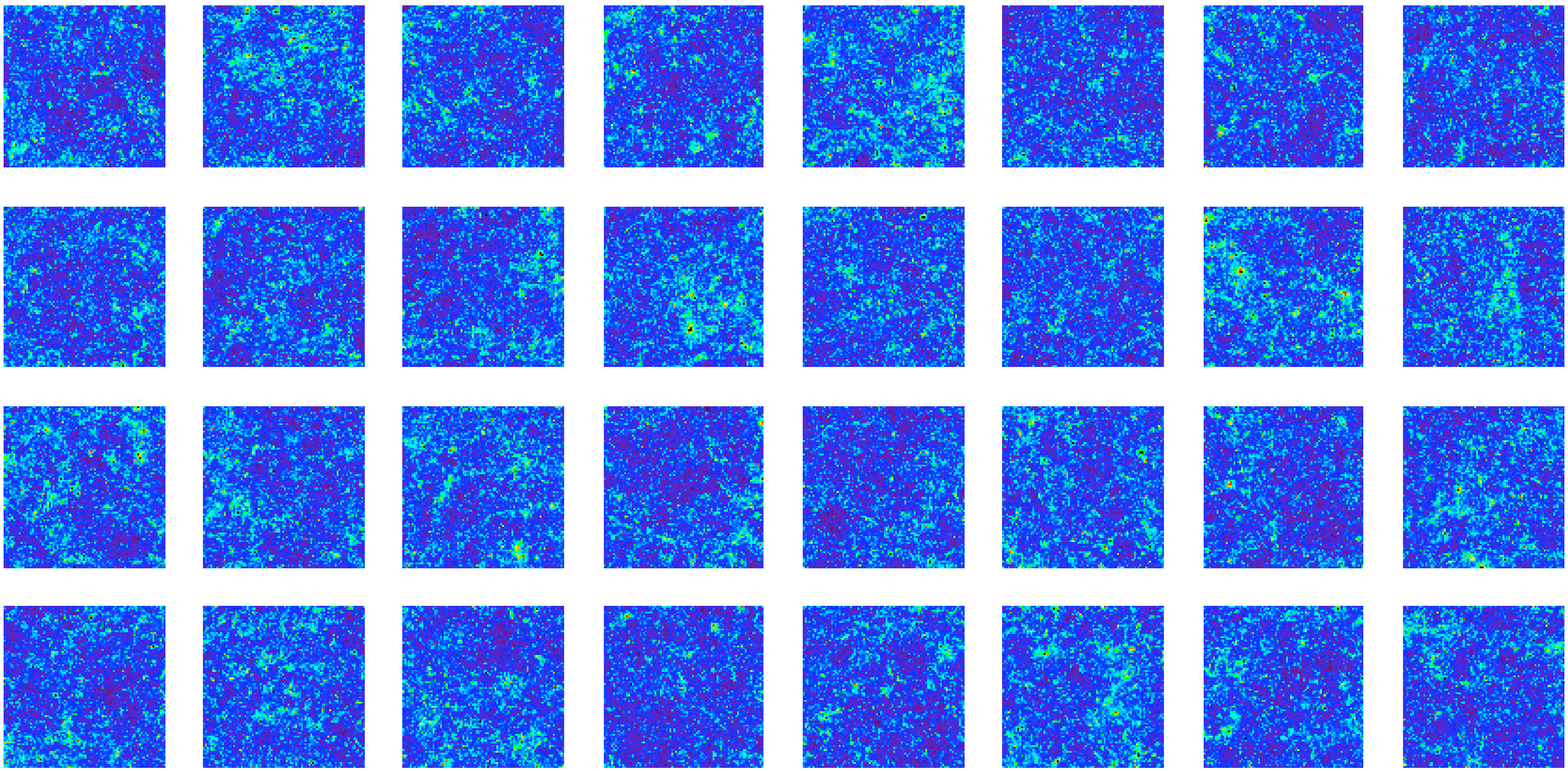
$$\vec{\theta}_n = -\sum_{p=1}^{n-1} \frac{r(\chi_n - \chi_p)}{r(\chi_n)} \nabla_\perp \psi_p + \vec{\theta}_1$$

$$\mathbf{A}_n = \mathbf{I} - \sum_{p=1}^{n-1} g(\chi_p, \chi_n) \mathbf{U}_p \mathbf{A}_p$$

$$U_{ij} \equiv \frac{\partial^2 \psi_p}{\partial x_i \partial x_j}$$

$\Omega_m = 0.357$ $\omega = -0.8$ $h = 0.64$ $n = 1.00$ $\sigma_8 = 0.88$ $\tau = 0.15$

(with Chris Vale)



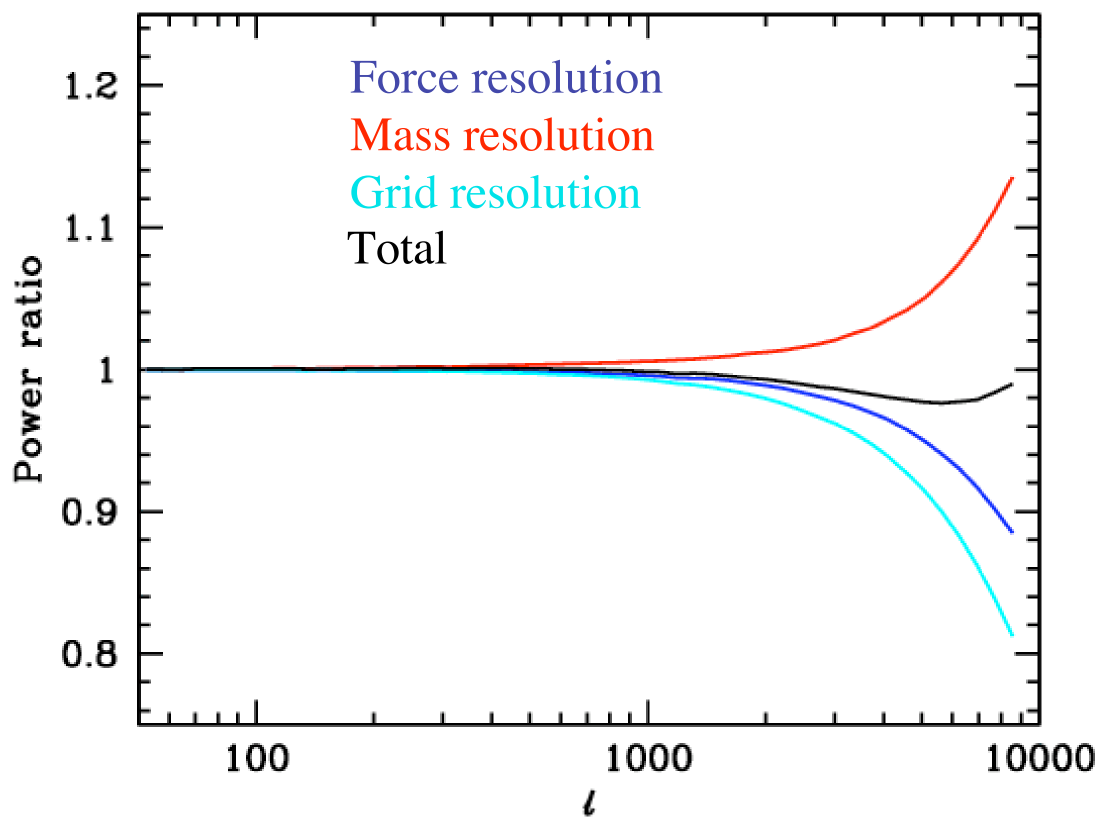
32 convergence maps, 3^o on a side

http://mwhite.berkeley.edu/Lensing/

Numerical Effects: Resolution

- The effects of finite resolution are understood
- We can predict the cost to achieve a given accuracy

C_l convergence study

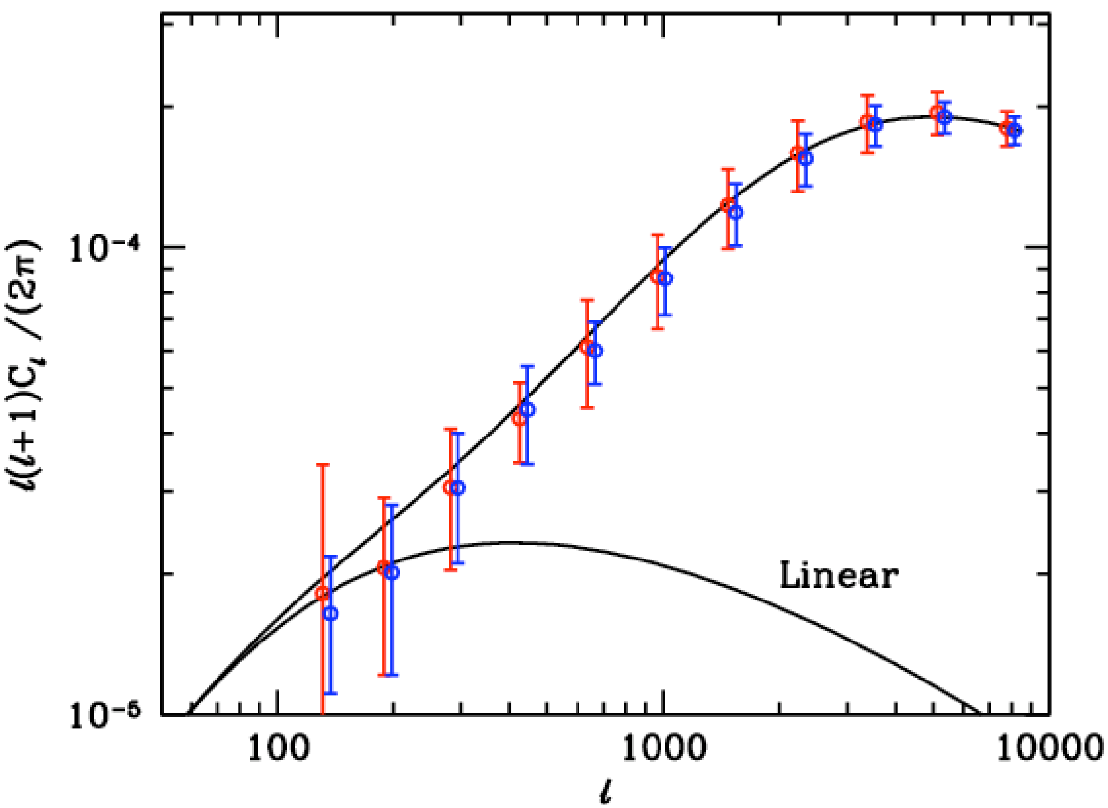


(from Vale & White 2003)

All "theory" is simulation based

(but different routes to final answer)

- Semi-analytic fits to power spectra or halo profiles and mass functions vs. direct simulation.
- For 2-pt and 3-pt functions the agreement is good! (Good enough?)
- Each method has different strengths and weaknesses.



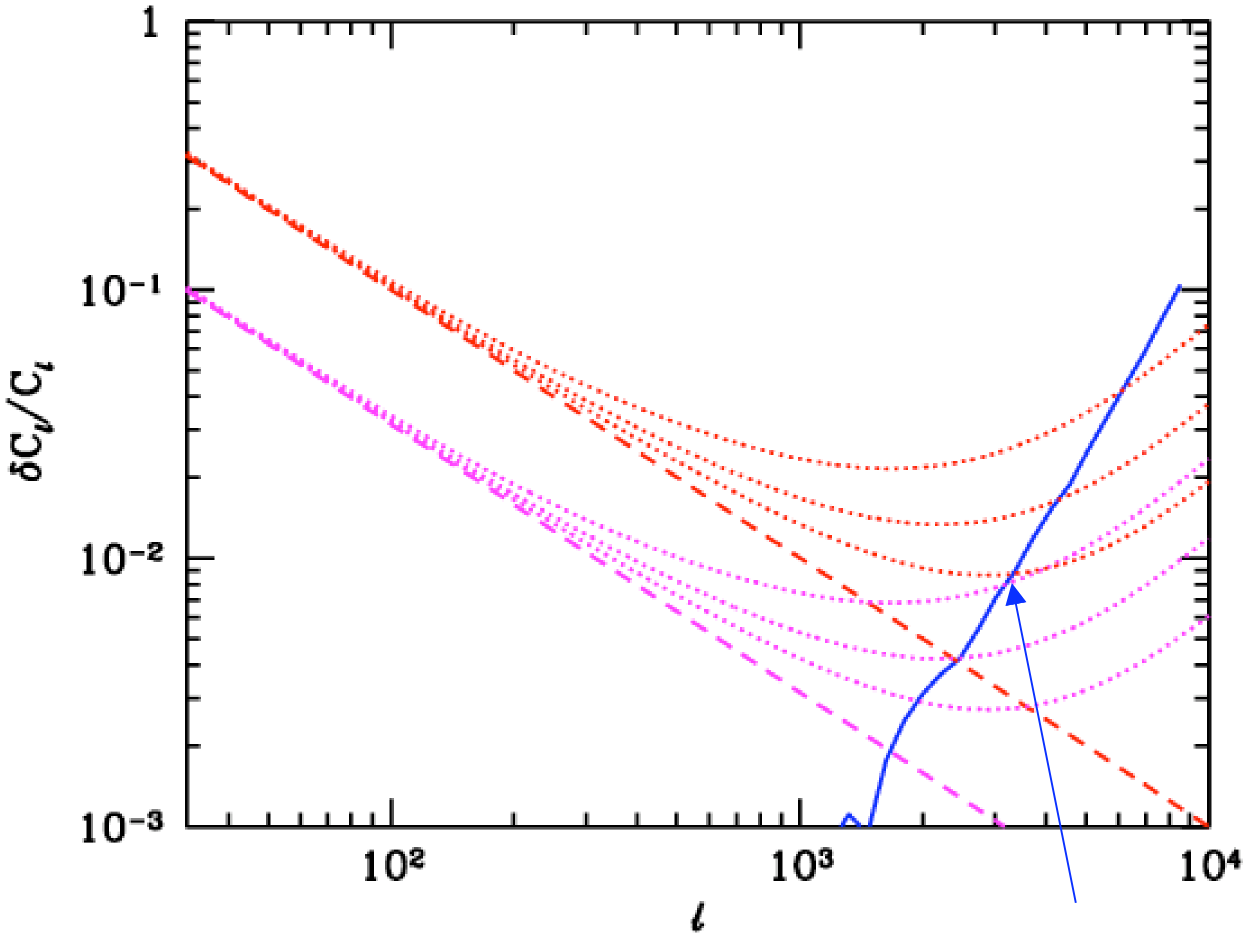
Beyond N-body

Gravitational lensing is "simple" because it involves only gravity, albiet non-linear gravity. However non-gravitational physics does become important on small scales:

- Baryonic cooling produces steep inner cusps in galaxies, leading to strong (extreme) lensing events.
- Contraction of baryons by cooling alters the potential in the surroundings, changing the lensing signal.
- Cooling alters the profiles of sub-halos, affecting lensing.

It is difficult to model these effects accurately at present, but we can make toy models to guesstimate the size of the effects.

Baryonic cooling



Using a simple model of cooling and adiabatic contraction can guess how cooling affects lensing C_l for sources at z~1.

(Cooling)-(no cooling)

The ultimate source screen …

Lensing of the CMB

(see talk by A. Amblard!)

Of course galaxies aren't the only source of (lensed) light in the universe. Any screen will do. Large-scale structure will lens the CMB anisotropy.
Since we don't know the "shape" of the CMB *a priori* we need to use more statistical information.

Quadratic estimator:
if I have a field x with $\langle x.x \rangle = C = C_0 + p\, C_1$ then a quadratic estimator of p is generically $Q_{ij}\, x_i\, x_j$
Requiring this to be unbiased and minimum variance for Gaussian x gives

$$\widehat{p} \propto x^T C^{-1} C_1 C^{-1} x + \cdots$$

Lensing of the CMB (contd)

Now consider the CMB, lensed

$$T(\hat{n}) = \widetilde{T}(\hat{n} - \nabla\Phi) \simeq \widetilde{T}(\hat{n}) - \nabla\Phi \cdot \nabla\widetilde{T}$$

The correlation function will depend on Φ, allowing us to make a quadratic estimator assuming everything is Gaussian and the deflection angle is small.

$$\hat{\kappa} \sim \nabla \cdot \widehat{\nabla\Phi} \sim \nabla \cdot [T_w \nabla T_w]$$

(Hu; Hirata & Seljak)

We should be able to detect this effect with upcoming experiments!

Unfortunately the sky is not Gaussian enough and the deflections not small enough to make this a 1% measurement.

The End

Recent reviews

- Mellier, 1999, ARA&A, 37, 127
- Bartelmann & Schneider, 2001, Phys. Rep., 340, 291
- Hoekstra, Yee & Gladders, 2002, New Astronomy Reviews, 46, 767
- Refregier, et al., 2003, ARA&A, 41, 645
- van Waerbeke & Mellier, astro-ph/0305089